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Quantum Networks
NSF Engineering Research Center

Principles of Quantum Networks

Don Towsley
– University of Massachusetts Amherst

Matheus Andrade
— University of Massachusetts Amherst

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CQN Winter School on Quantum Networks

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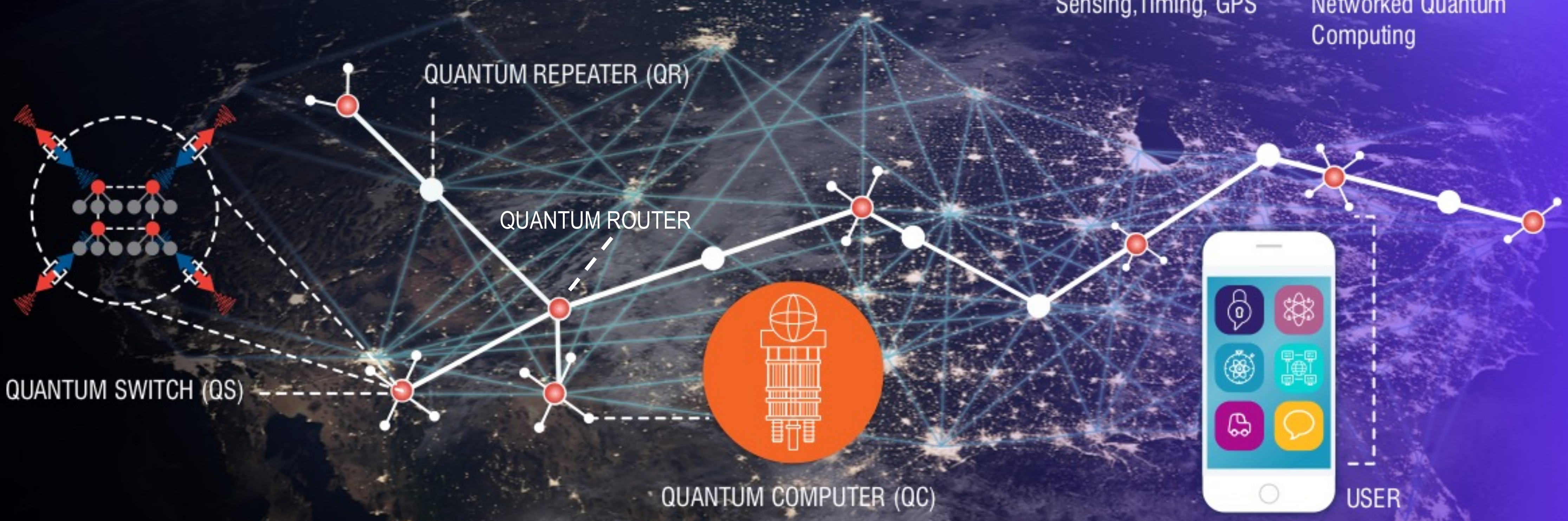
Outline

- Introduction
- Classical vs. quantum networks
- Capacity and resource allocation
- Network management and quantum tomography
- Summary

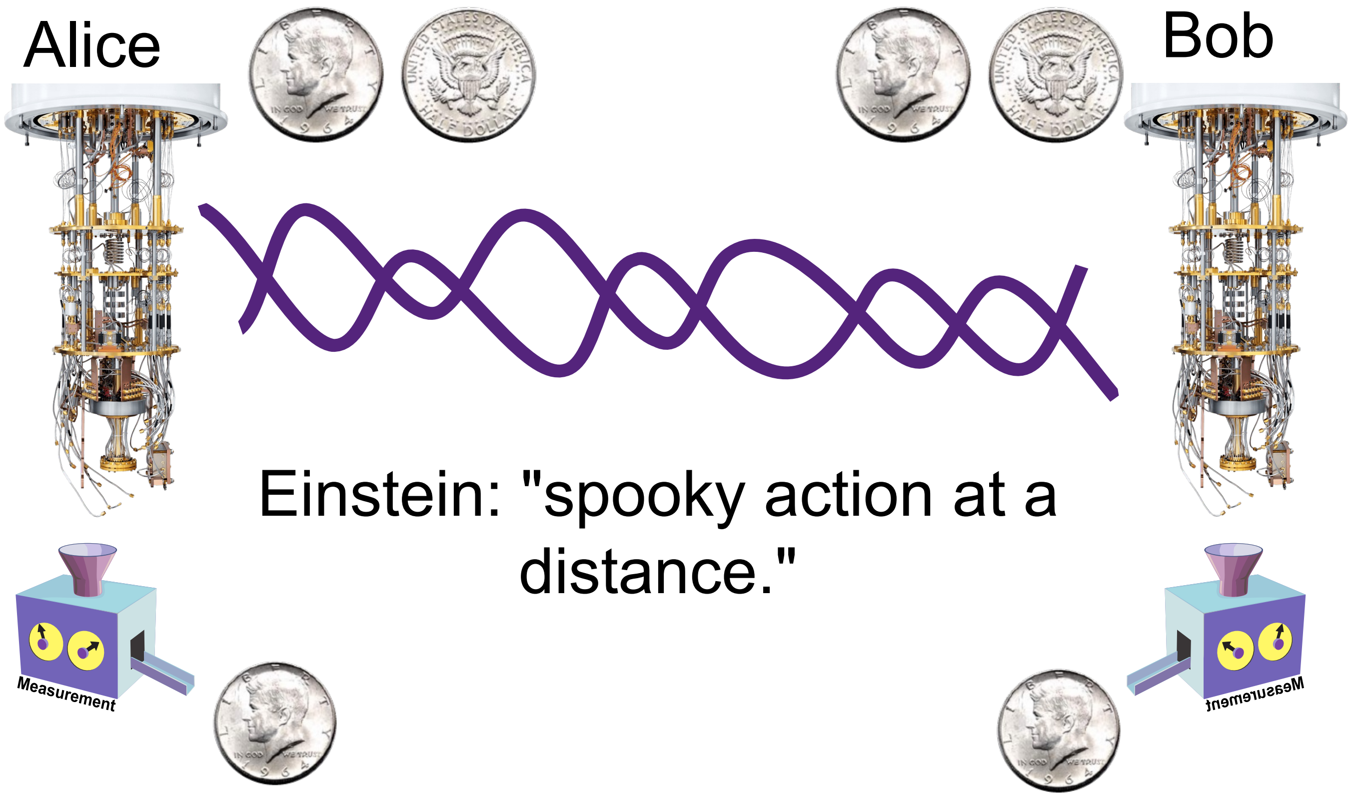
The Quantum Internet

Vision: Quantum network enabling full quantum connectivity between multiple user groups.



Key ingredient

Quantum entanglement, *aka Bell state*, between pair of remote quantum processors



Einstein: "spooky action at a distance."

Bell state: $\frac{|0_A 0_B\rangle + |1_A 1_B\rangle}{\sqrt{2}}$

Nobel prize, Physics, 2022: A. Aspect, F. Clauser, A. Zeilinger

https://pubs.acs.org/doi/10.1021/cen-10036-scicon3

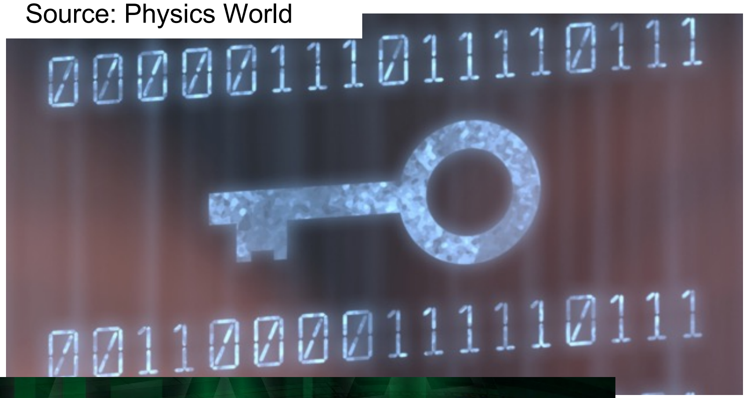
Why Quantum Internet?

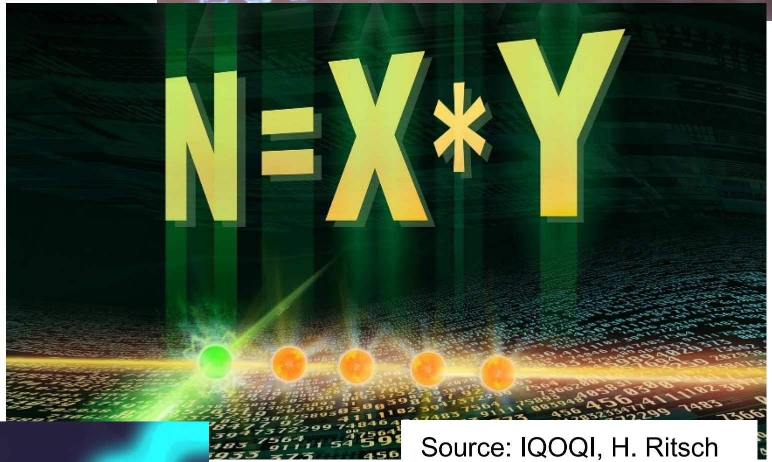
Cryptography, security – quantum key distribution (QKD)

Distributed quantum computing – breaking web security, solving hard problems

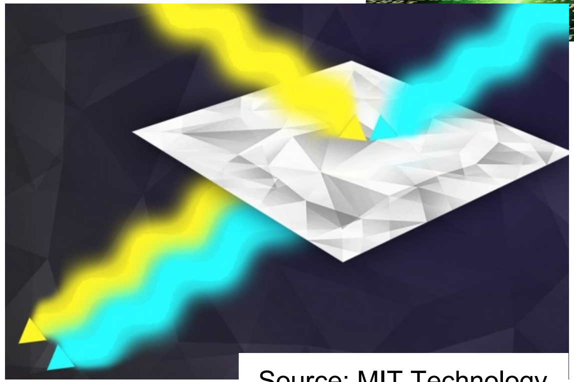
High resolution sensing – exploring the universe

Source: Physics World





Source: IQOQI, H. Ritsch



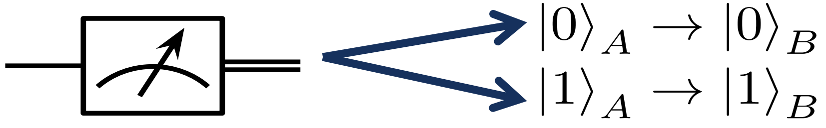
Source: MIT Technology

Bell state

- Bell state

$$\frac{|0_A 0_B\rangle + |1_A 1_B\rangle}{\sqrt{2}}$$

- Measuring Alice's qubit yields 0,1
  - if 0, measuring Bob's qubit yields 0
  - if 1, measuring Bob's qubit yields 1
  - can generate shared randomness across distances



- Key ingredient of quantum teleportation, QKD, and many other applications

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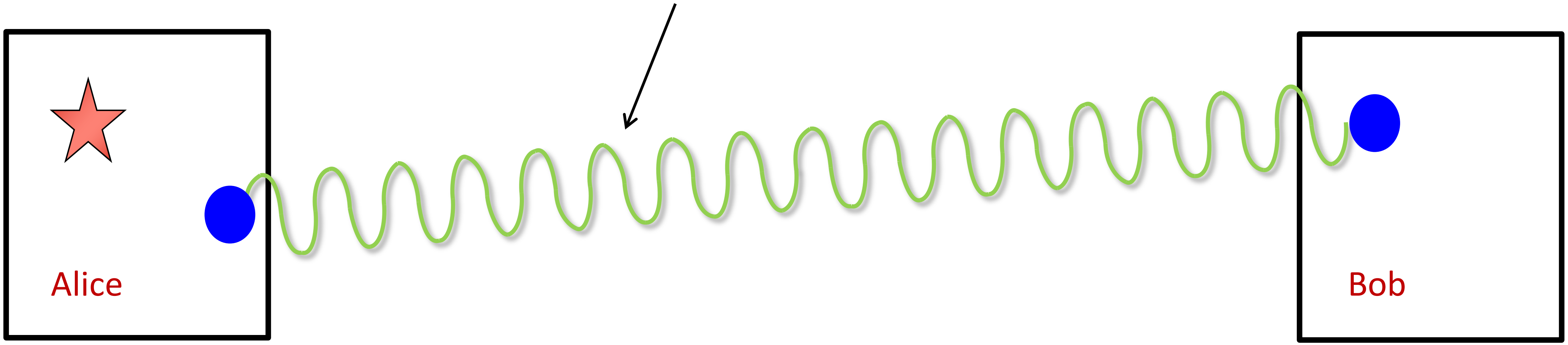
Quantum Teleportation

end-to-end entanglement $\frac{|0_A 0_B\rangle + |1_A 1_B\rangle}{\sqrt{2}}$



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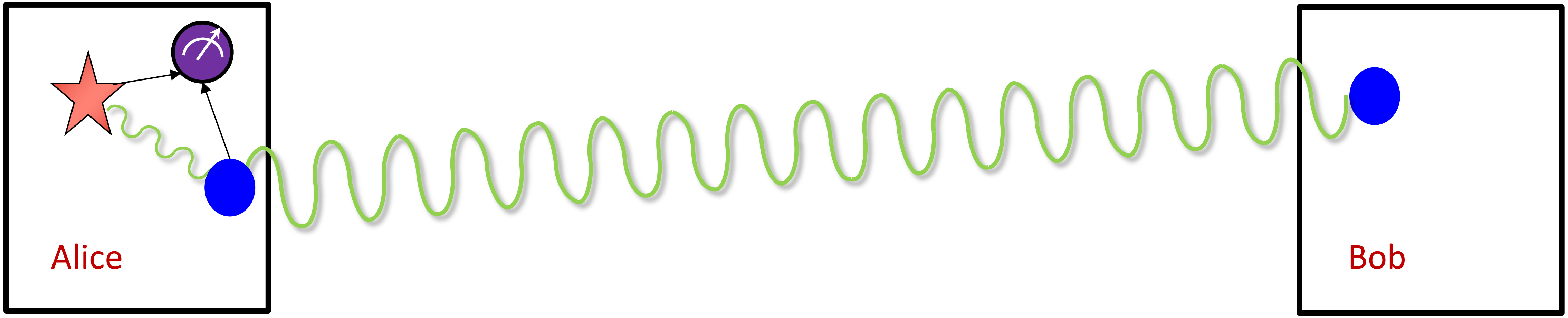
Teleportation



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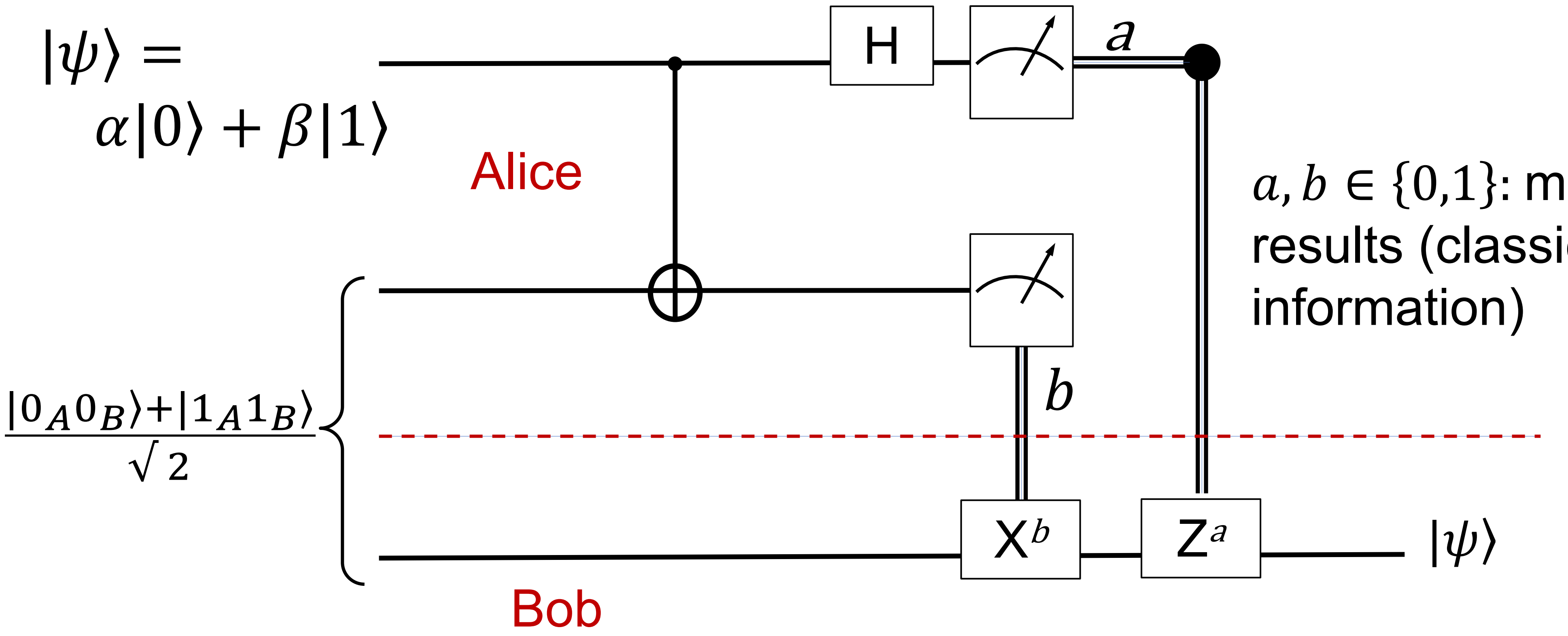
Teleportation

Teleportation circuit



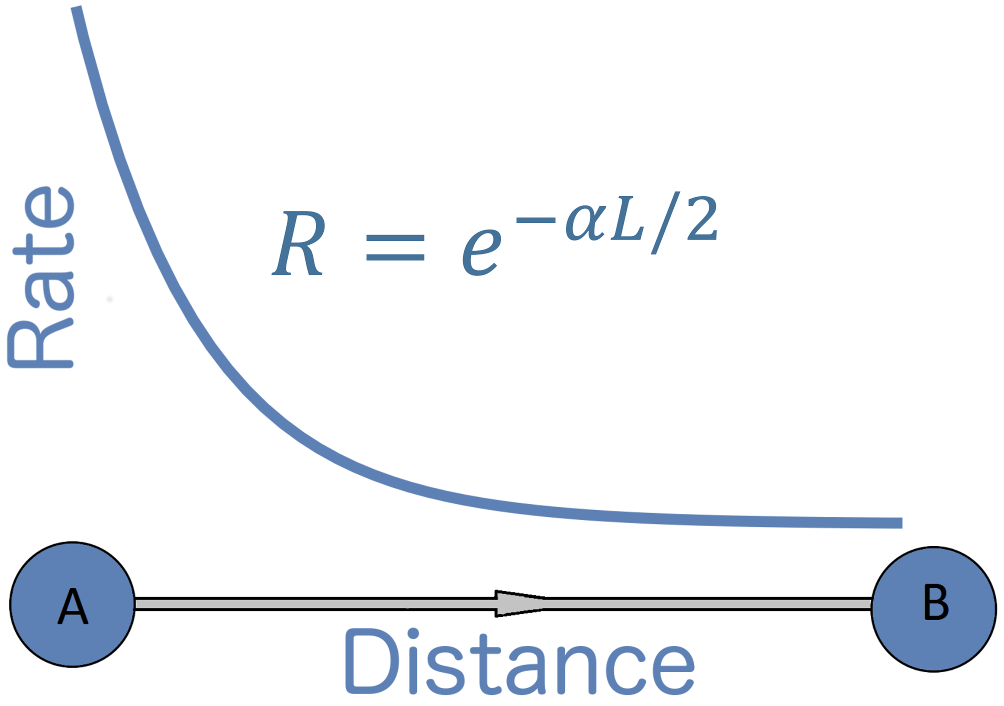
$a, b \in \{0,1\}$: measurement results (classical information)

Multipartite extension of Bell state

Greenberger–Horne–Zeilinger (GHZ) state

- n-partite GHZ state $|GHZ\rangle = \frac{|00\cdots 0\rangle + |11\cdots 1\rangle}{\sqrt{2}}$
- used in multiparty QKD, secret sharing, quantum sensing, …

Why is quantum communications so hard?



Rate decays exponentially with distance

Can we amplify signal?

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Why is it so hard?

Rate decays exponentially with distance

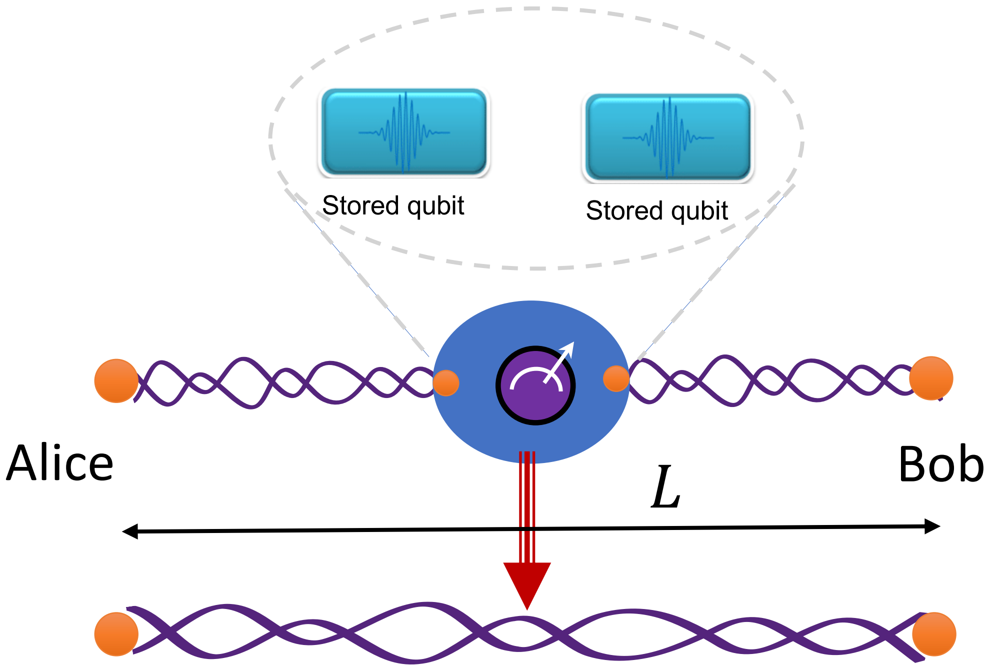
No cloning theorem!
Quantum signals cannot be copied

Quantum repeaters

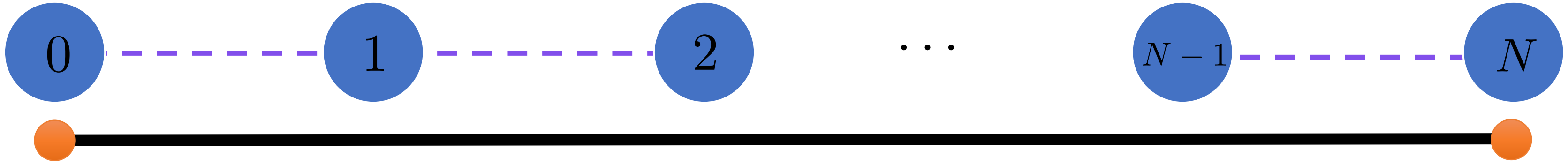
Quantum memories to store entanglement

Phase I: generate link level entanglement (Bell states)

Phase II: measurement propagates entanglements to ends



$$R = e^{-\alpha L/2}$$



$$R \propto e^{-\alpha L/N}$$

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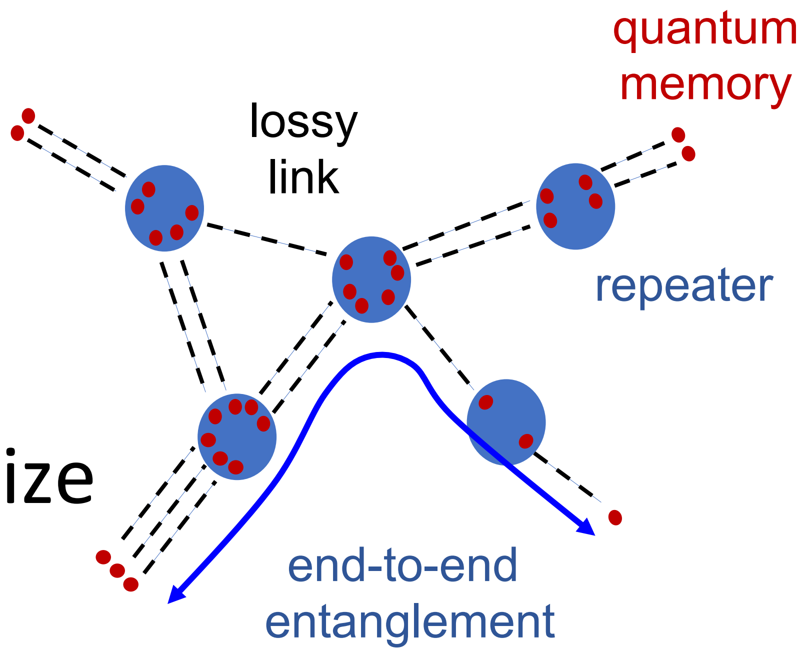
Quantum entanglement network

- Quantum switches with memories connected via lossy links
- Links generate entanglement
- Switches concatenate (measure) to realize end-to-end entanglement between end nodes



Quantum networking challenges

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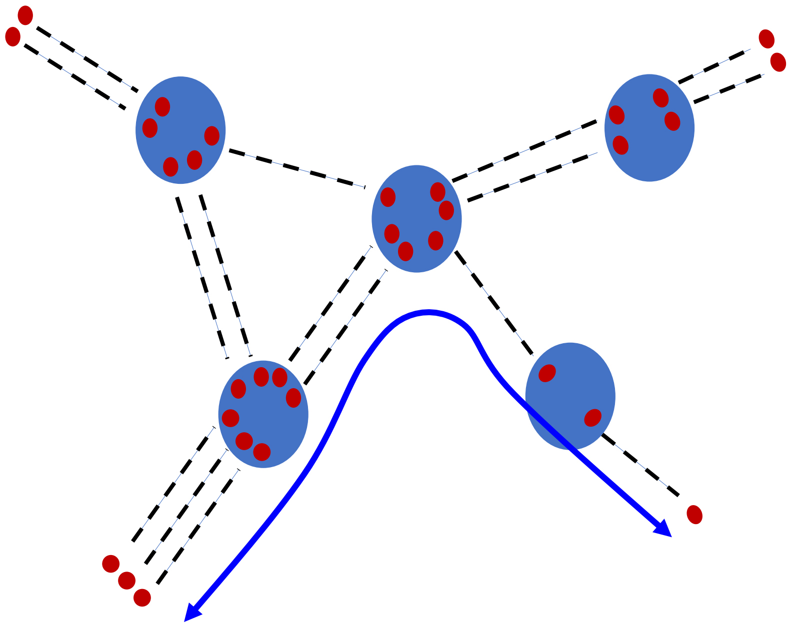
- Service to provide
  - entanglement distribution
  - direct quantum information transfer
- Noise!
- Who to serve
  - performance & resource allocation



- Network management
  - measurement & tomography
- Data, control plane design

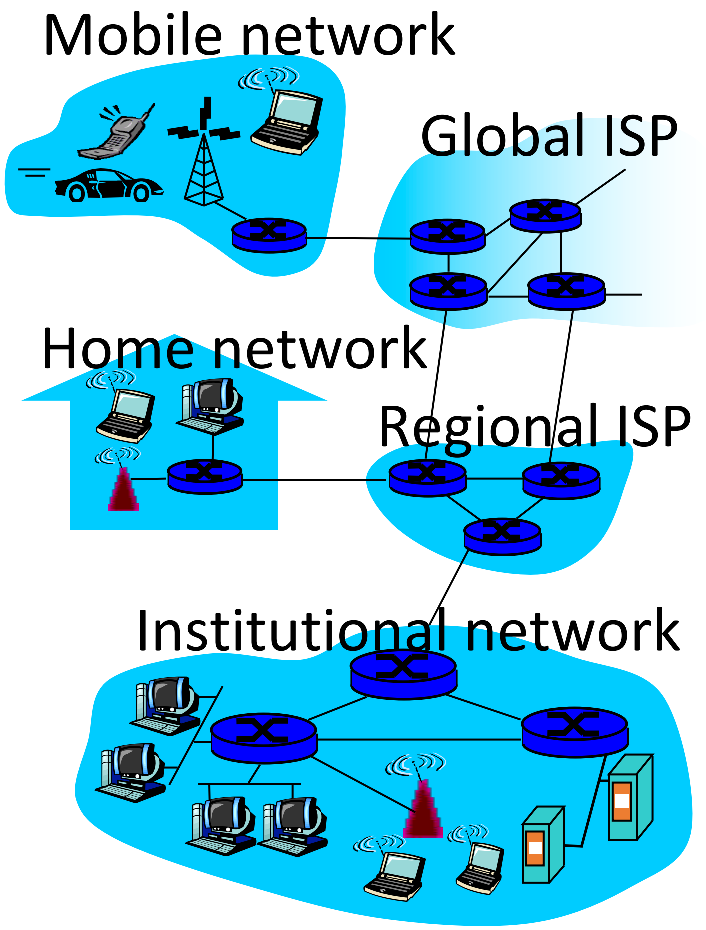
Classical vs. Quantum Networks

Outline

- Internet overview
- Network services, routing
- Switch/router design

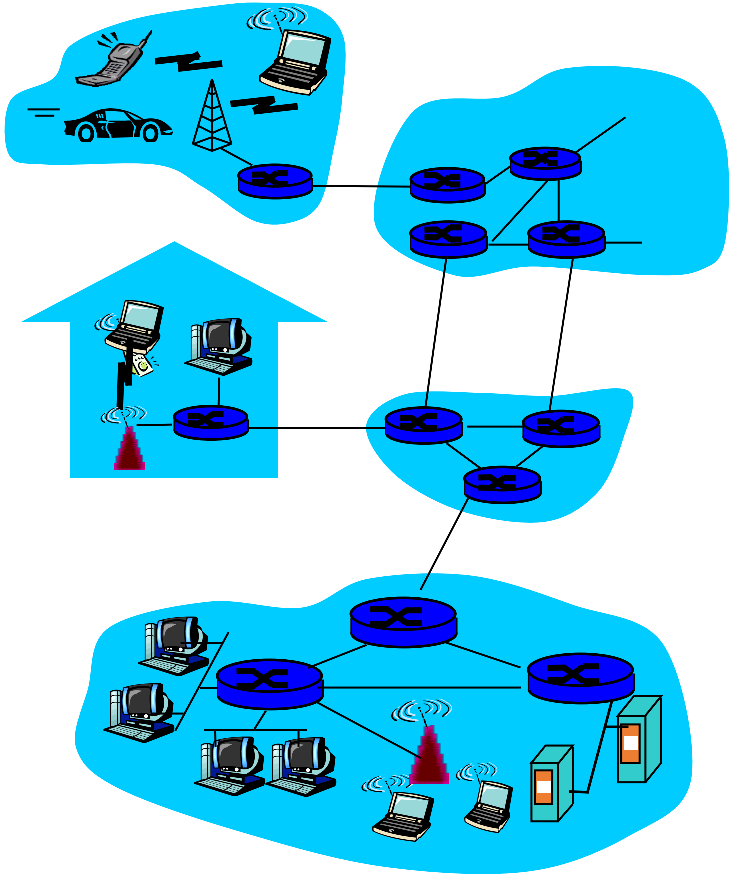
What's the Internet: "nuts and bolts" view

- *Internet:* "network of networks"
  - loosely hierarchical
  - public Internet versus private intranet
- *Protocols:* control sending, receiving of messages
  - e.g., TCP, IP, HTTP, Skype, Ethernet, WiFi
- Internet standards
  - RFC: Request for comments
  - IETF: Internet Engineering Task Force
  - IRTF: Internet Research Task Force



A closer look at network structure

- Network edge: applications and hosts
- Network core:
  - routers
  - network of networks
- Access networks
  - wired
  - wireless



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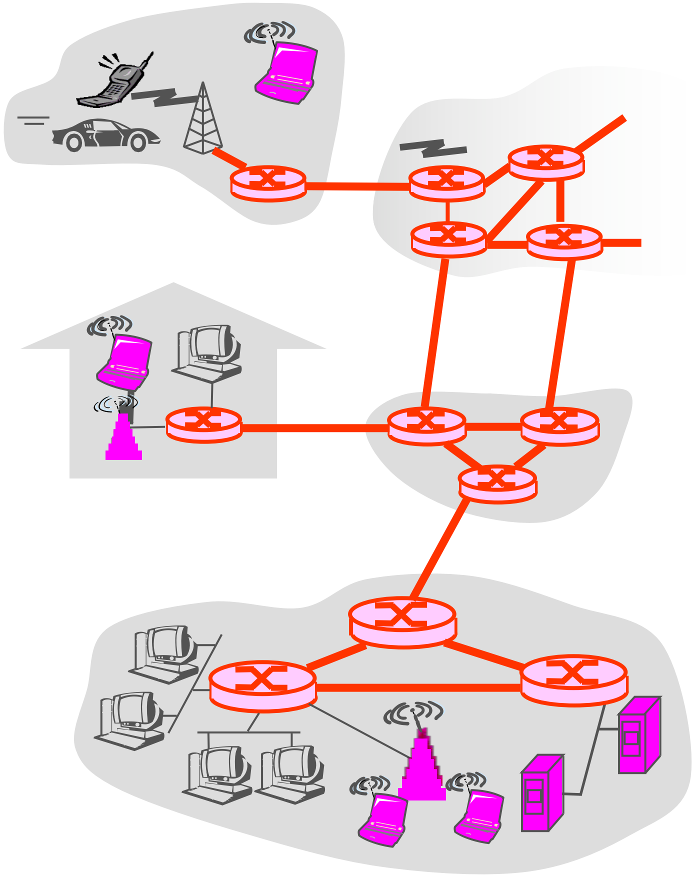
The network core

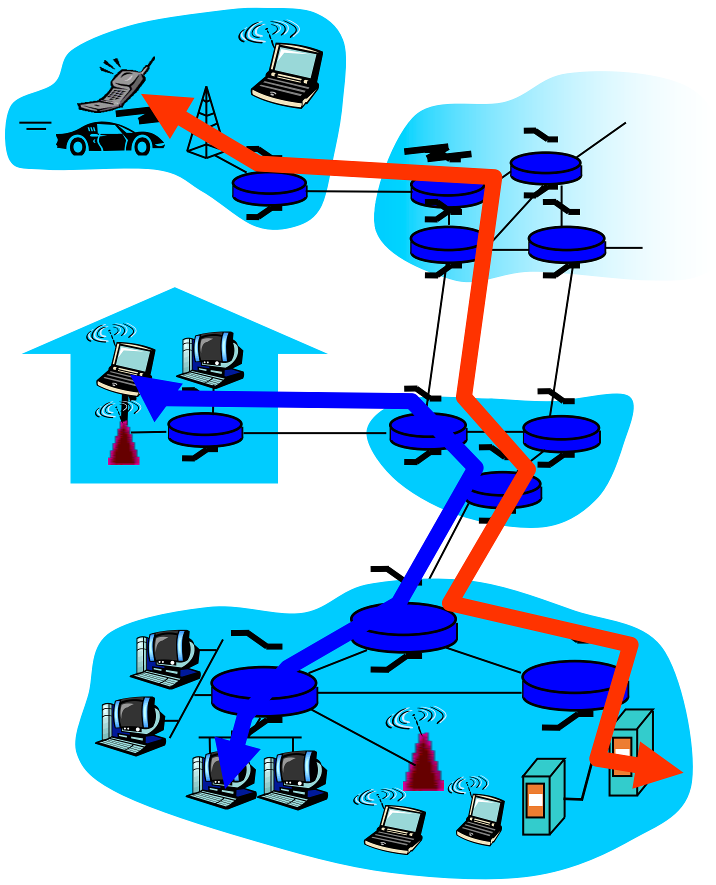
- Mesh of interconnected routers
- Fundamental question: how is data transferred through net?
  - circuit switching: dedicated circuit per call: telephone net
  - packet-switching: data sent thru net in discrete “chunks”



Network core: Circuit switching

End-end resources reserved for "call"

- Link bandwidth, switch capacity
- Dedicated resources: no sharing
- Circuit-like (guaranteed) performance
- Call setup required



Network core: Packet switching

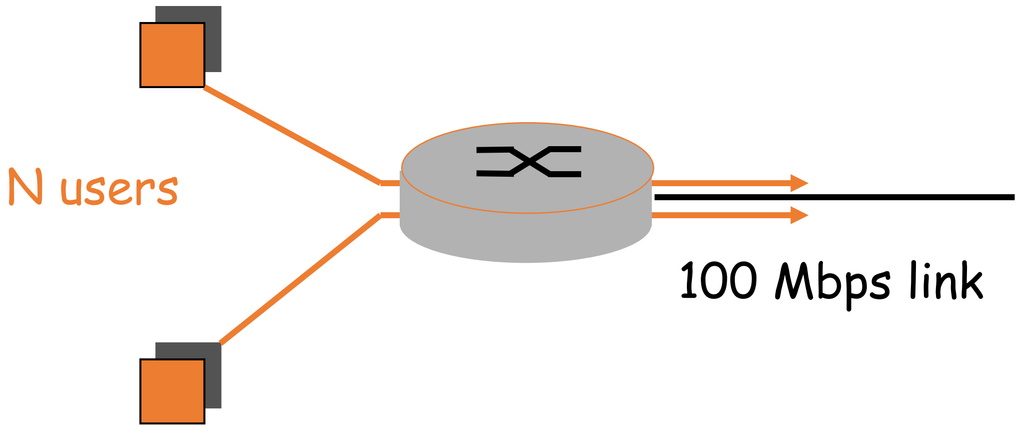
Each end-end data stream divided into *packets*

- User A, B packets *share* network resources
- Each packet uses full link bandwidth
- Resources used *as needed*

- Resource contention
- Aggregate resource demand can exceed amount available
- Congestion: packets queue, wait to use link
- Store and forward: packets move one hop at a time
  - transmit over link
  - wait turn at next link

Packet switching versus circuit switching

- 100 Mb/s link
- each user:
  - 10 Mb/s when "active"
  - active 10% of time



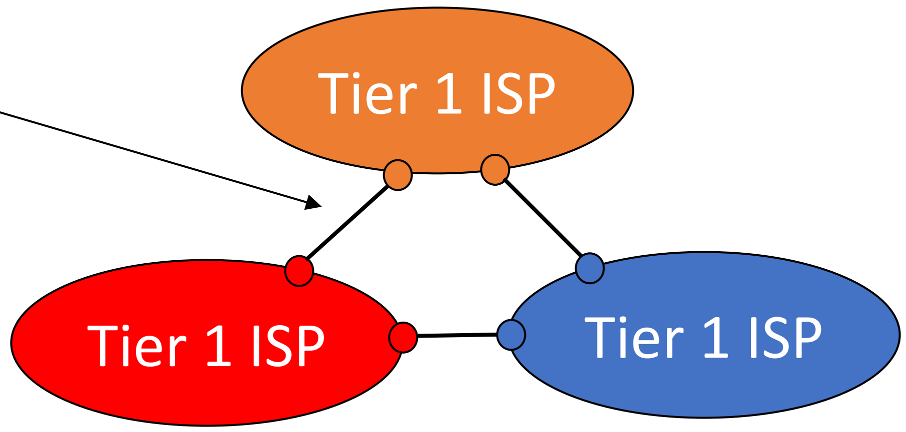
- Circuit-switching:
  - 10 users
- Packet switching:
  - with 35 users, probability > 10 active less than .0004

Packet switching allows more users to use network!

Internet structure: network of networks

- Roughly hierarchical
- At center: "tier-1" ISPs (e.g., Verizon, Sprint, AT&T, Level 3), national/international coverage
  - treat each other as equals

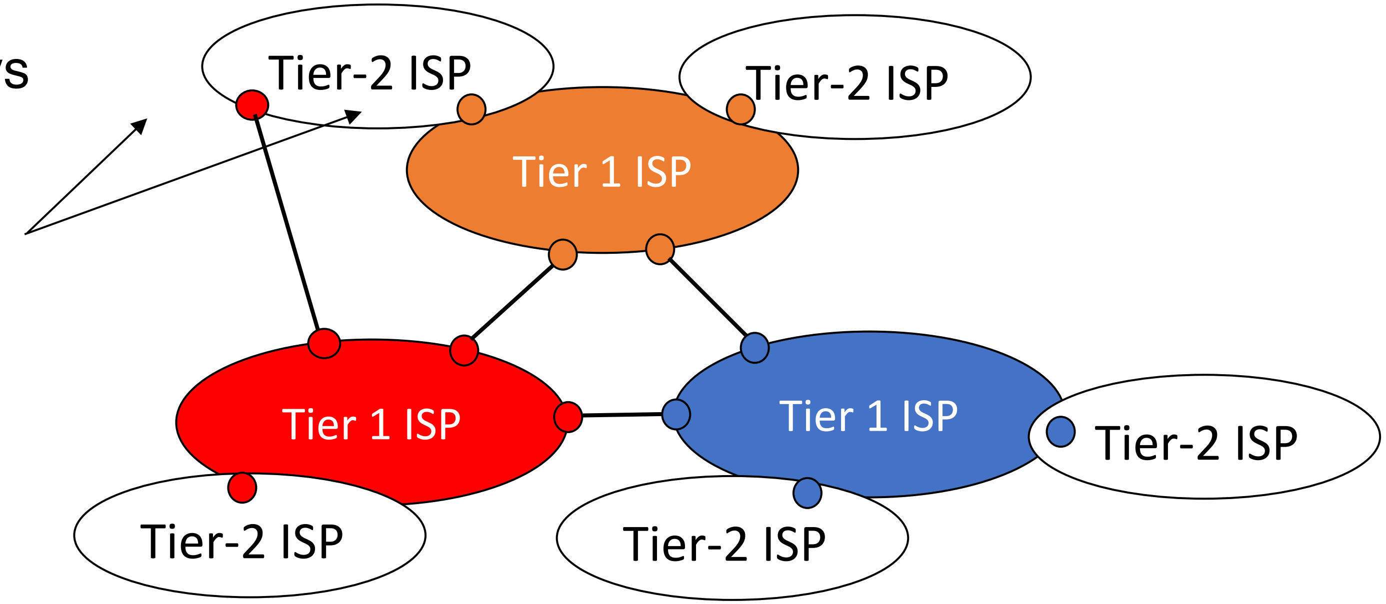
Tier-1 providers interconnect (peer) privately



Internet structure: network of networks

- "Tier-2" ISPs: smaller (often regional) ISPs
  - connect to one or more tier-1 ISPs, possibly other tier-2 ISPs

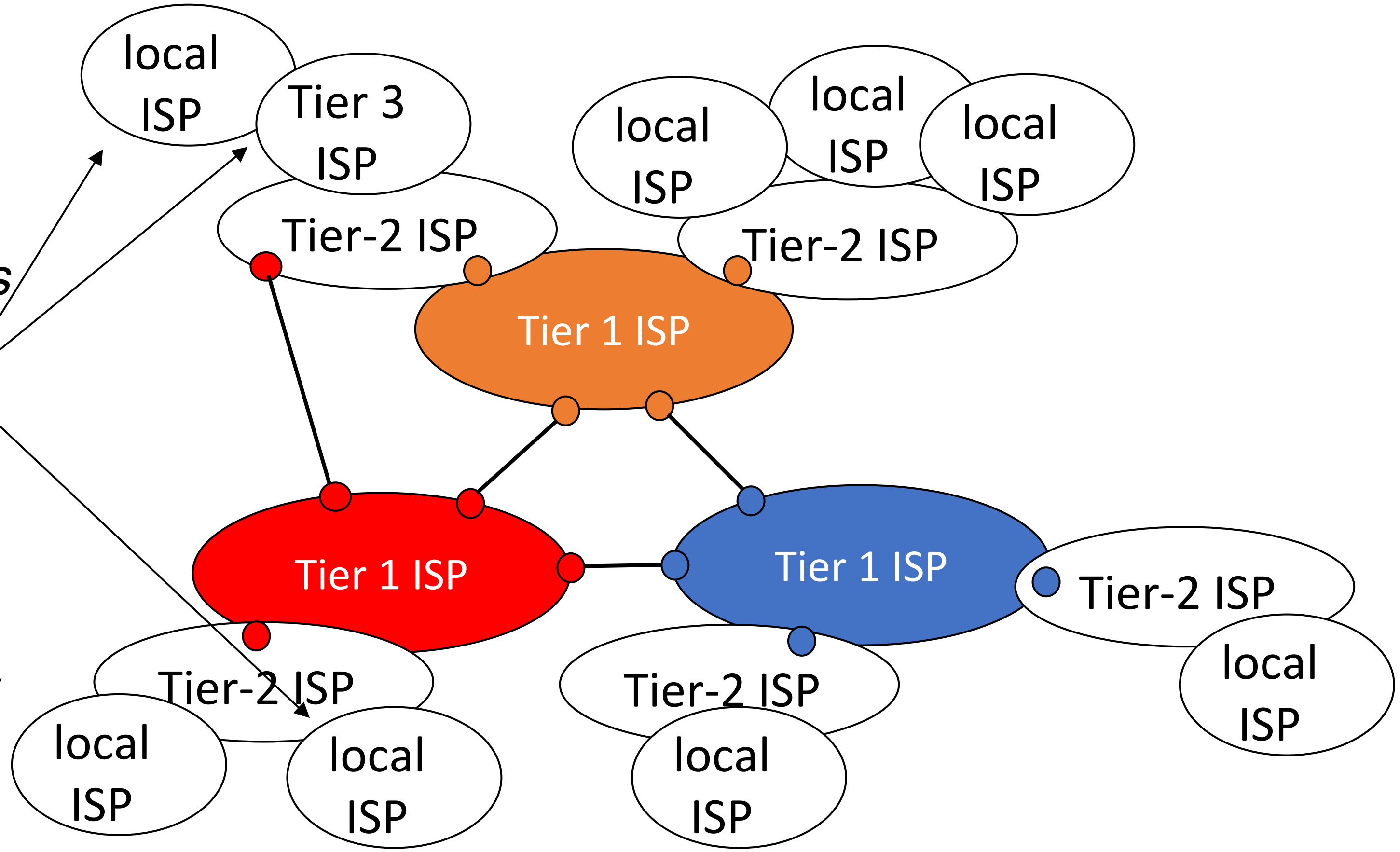
- ❑ tier-2 ISP pays tier-1 ISP for connectivity to rest of Internet
- ❑ tier-2 ISP is *customer* of tier-1 provider



Internet structure: network of networks

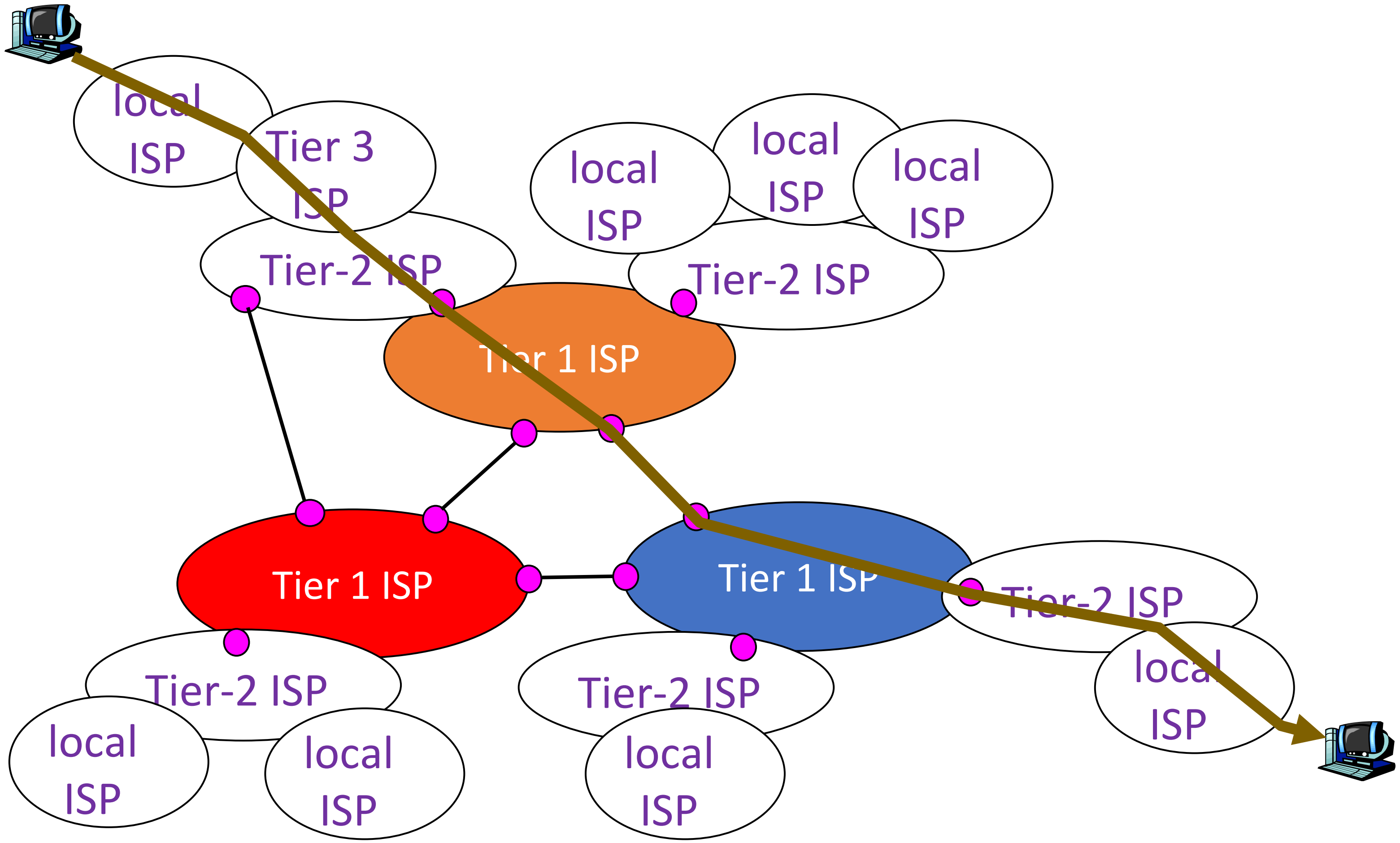
- "Tier-3" ISPs and local ISPs
  - last hop ("access") network (closest to end systems)

Local and tier- 3 ISPs are *customers* of higher tier ISPs connecting them to rest of Internet



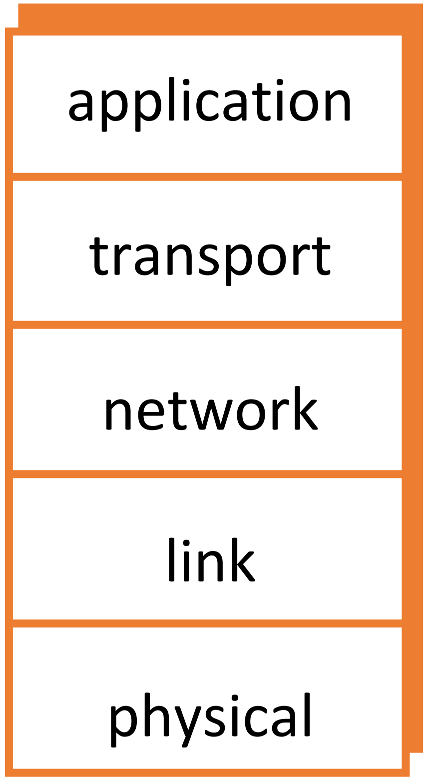
Internet structure: network of networks

- a packet passes through many networks!



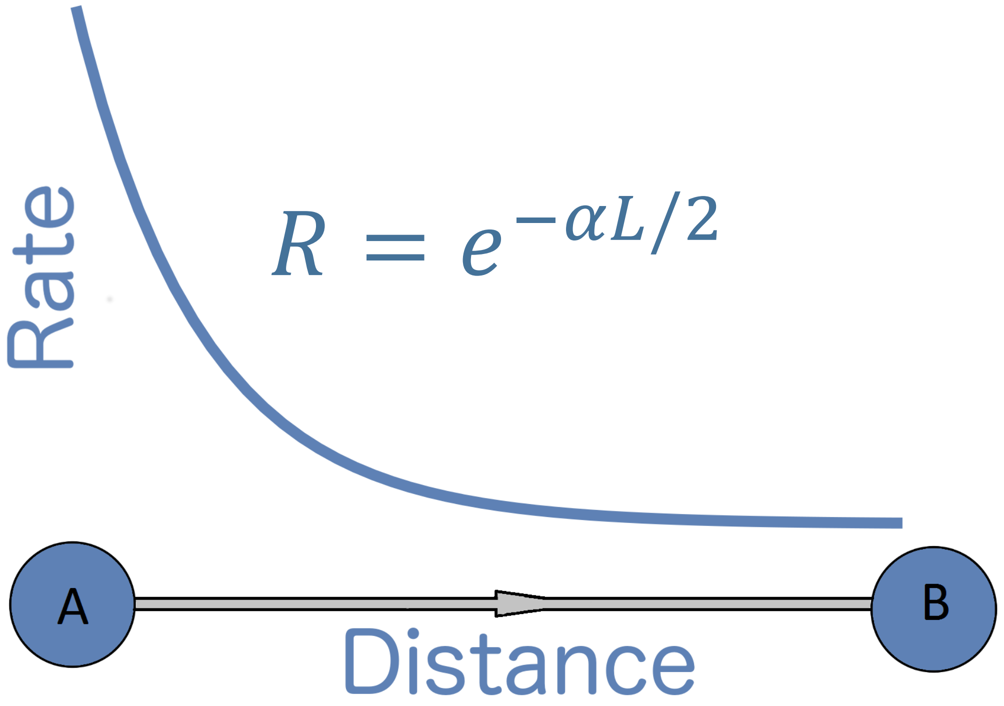
Internet protocol stack

- Application: supporting network applications
  - scp, smtp, https
- Transport: host-host data transfer
  - tcp, udp
- Network: routing of packets from source to destination
  - ip, routing protocols
- Link: data transfer between neighboring network elements
  - ppp, ethernet
- Physical: bits “on the wire”



Quantum Networks

Why is quantum communications so hard?



Rate decays exponentially with distance

No cloning theorem precludes copy and amplification

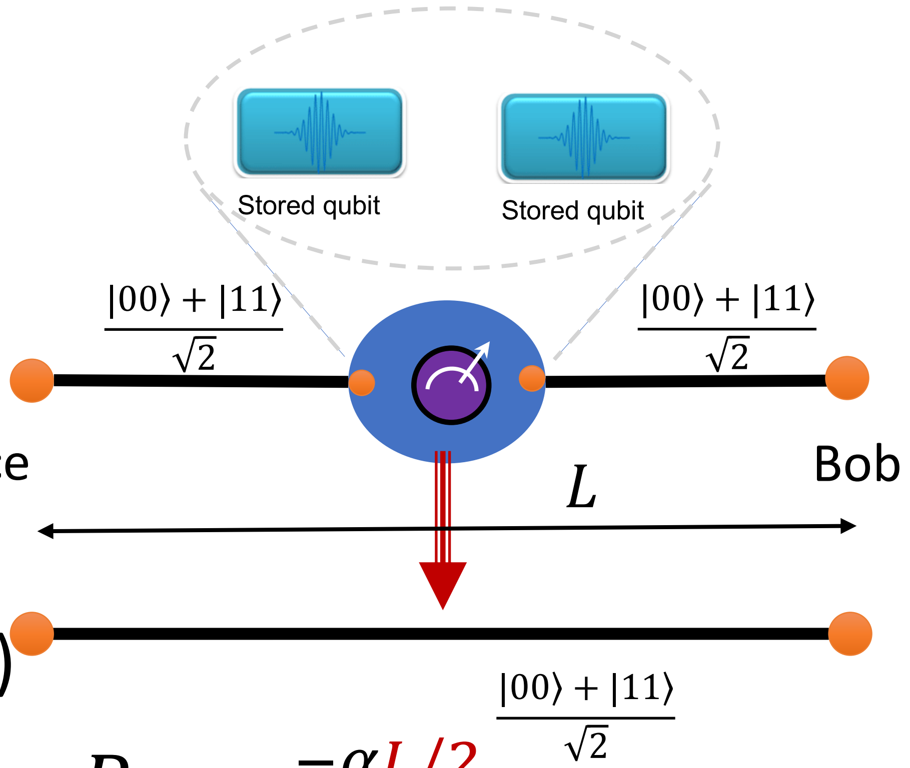
Quantum repeaters

Quantum memories to store qubits

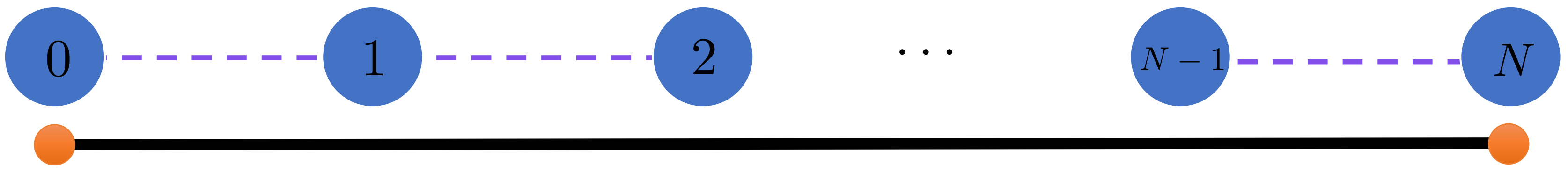
Phase I: generate link Bell states (entanglement)

Phase II: propagate entanglements

entanglement swap (Bell state measurement)



$$R = e^{-\alpha L/2}$$



$$R \propto e^{-\alpha L/N}$$

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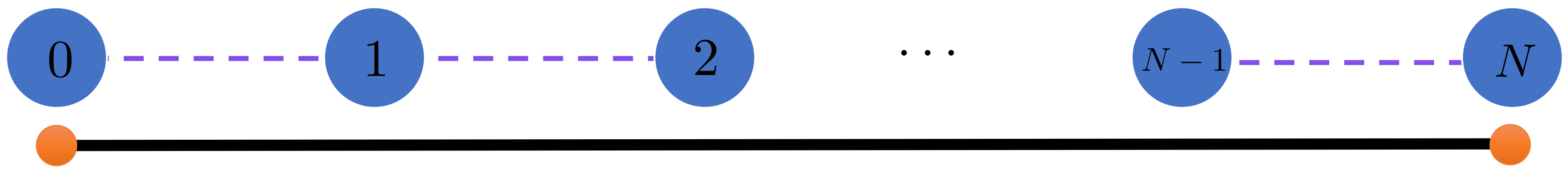
Repeater chain



- Infinite memory ⇒ distance independent entanglement rate

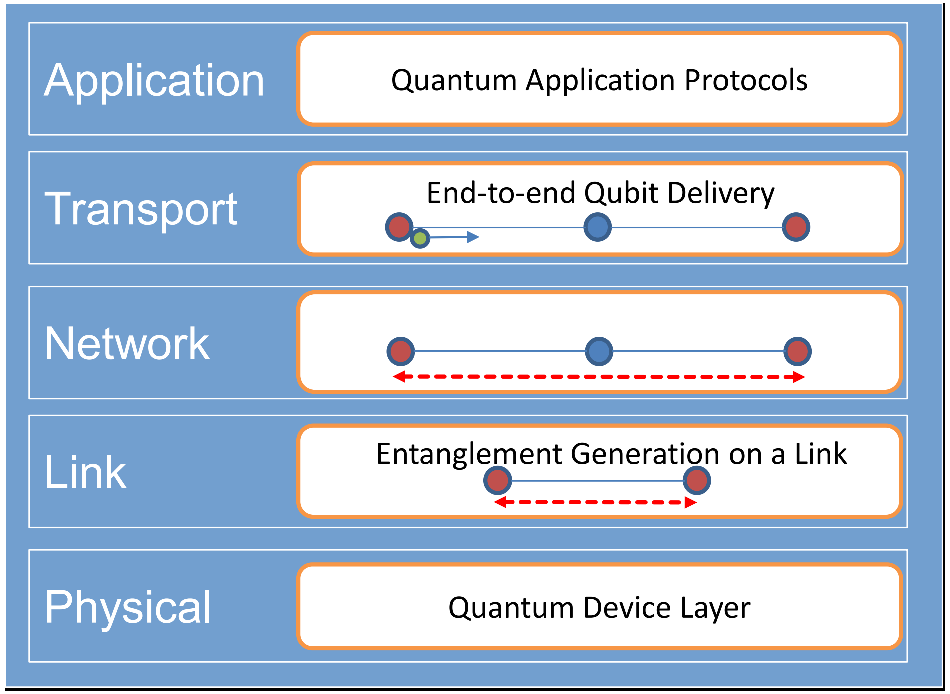
$$R \propto e^{-\alpha L/N}$$

- Finite (one) memory ⇒ exponential decay in entanglement rate as function of L

$$R \propto e^{-\alpha L}$$

Quantum Internet

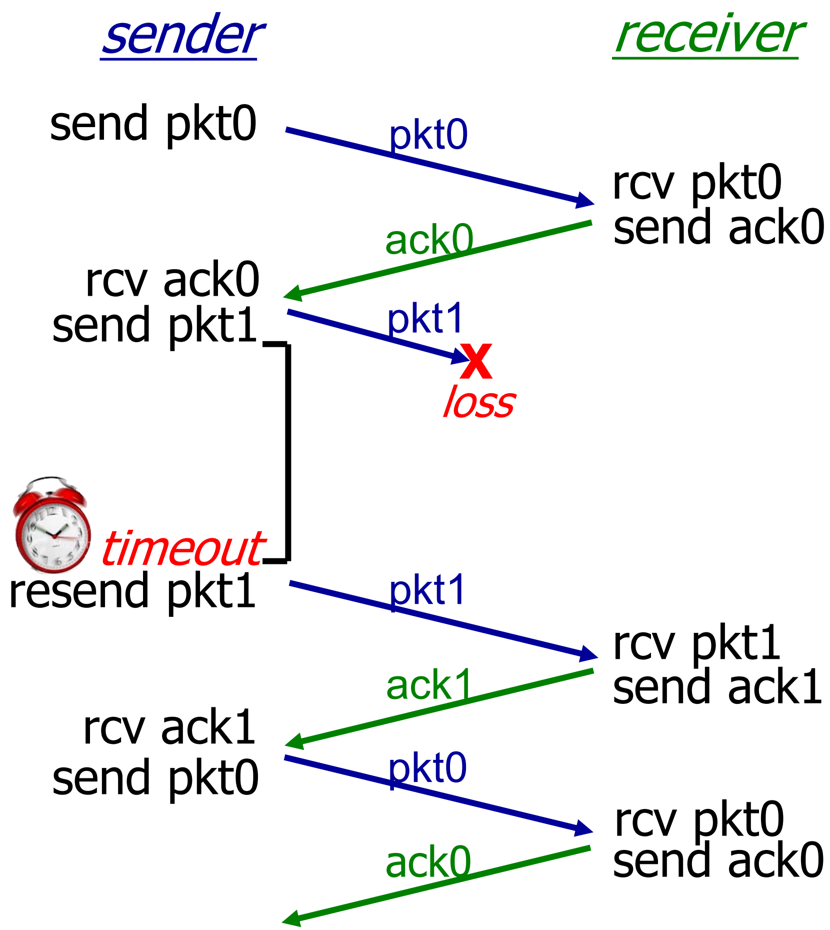
- Application: supporting network applications
- Transport: host-host quantum data transfer
  - qtcp, qudp
- Network: entanglement generation between end nodes
  - qip, path selection protocols
- Link: link-level entanglement generation
- Physical: photons “on the wire”



Stephanie Wehner et al.

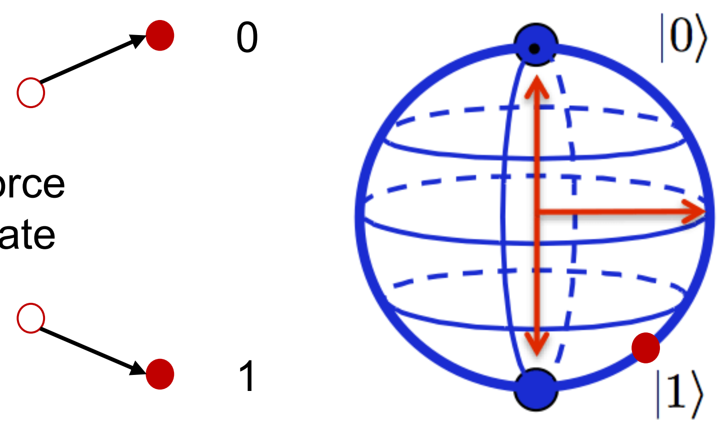
Reliable communications (classical)

- Error models:
  - bit flips, erasures
  - *dropped packets*
- Recovery schemes
  - error detection/correction codes
  - packet retransmission
    - *relies on cloning!*

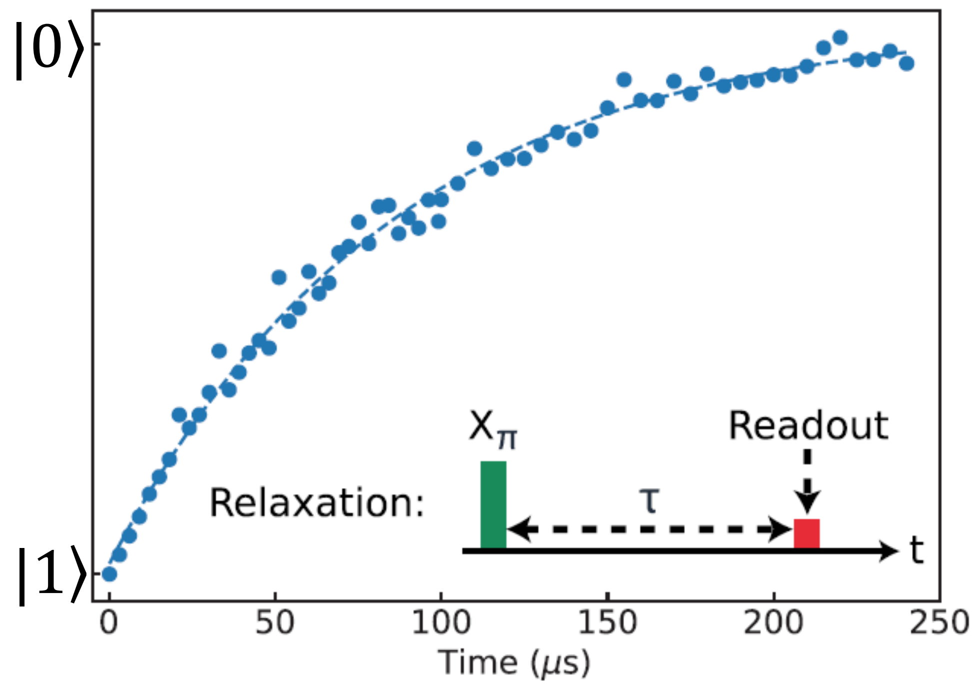


Quantum challenge

- Qubits not self protected against smallest perturbation

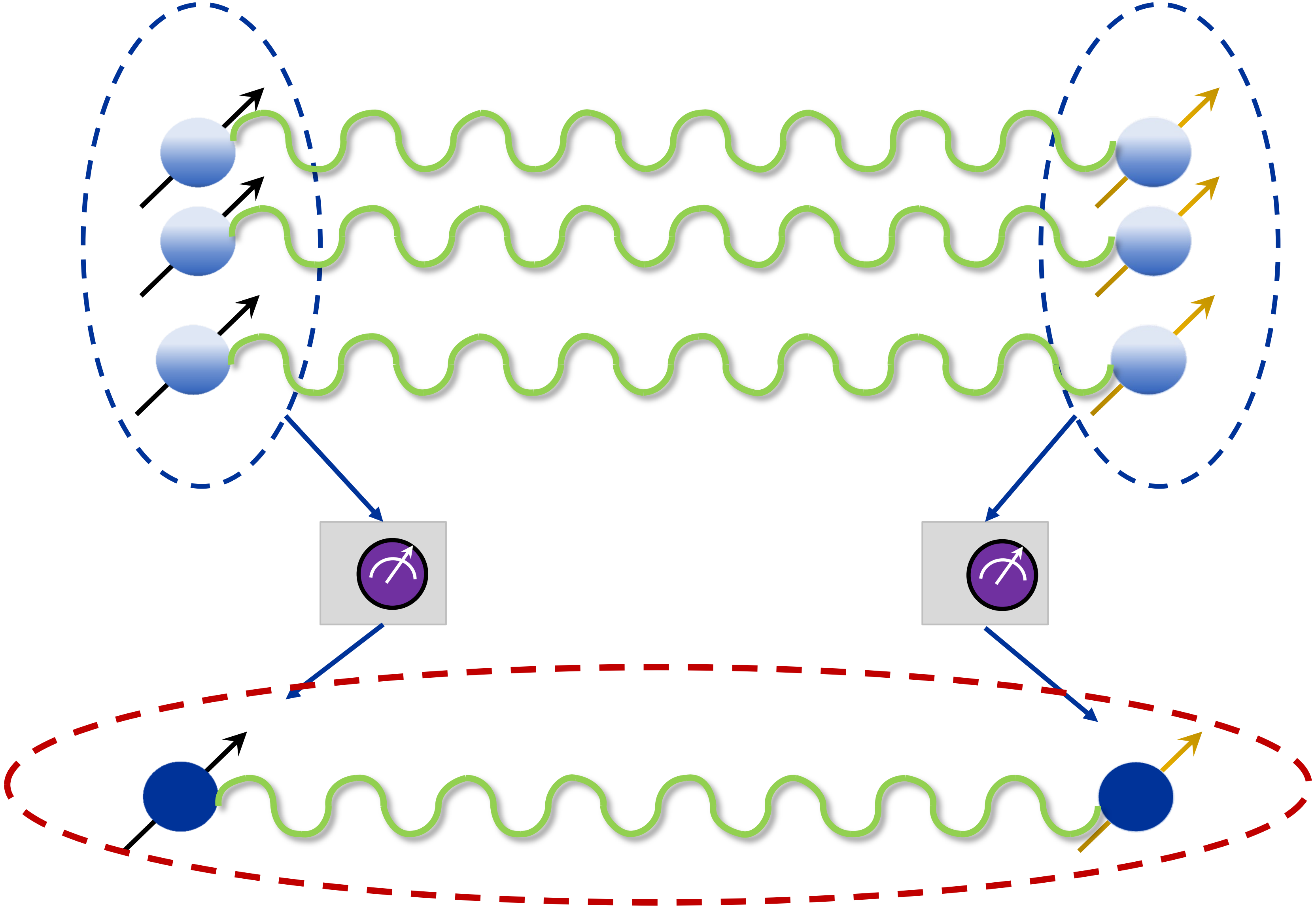


- Qubits have limited coherence times



P. Krantz, etal, Appl. Phys. Rev. 6, 021318 (2019)

Entanglement purification



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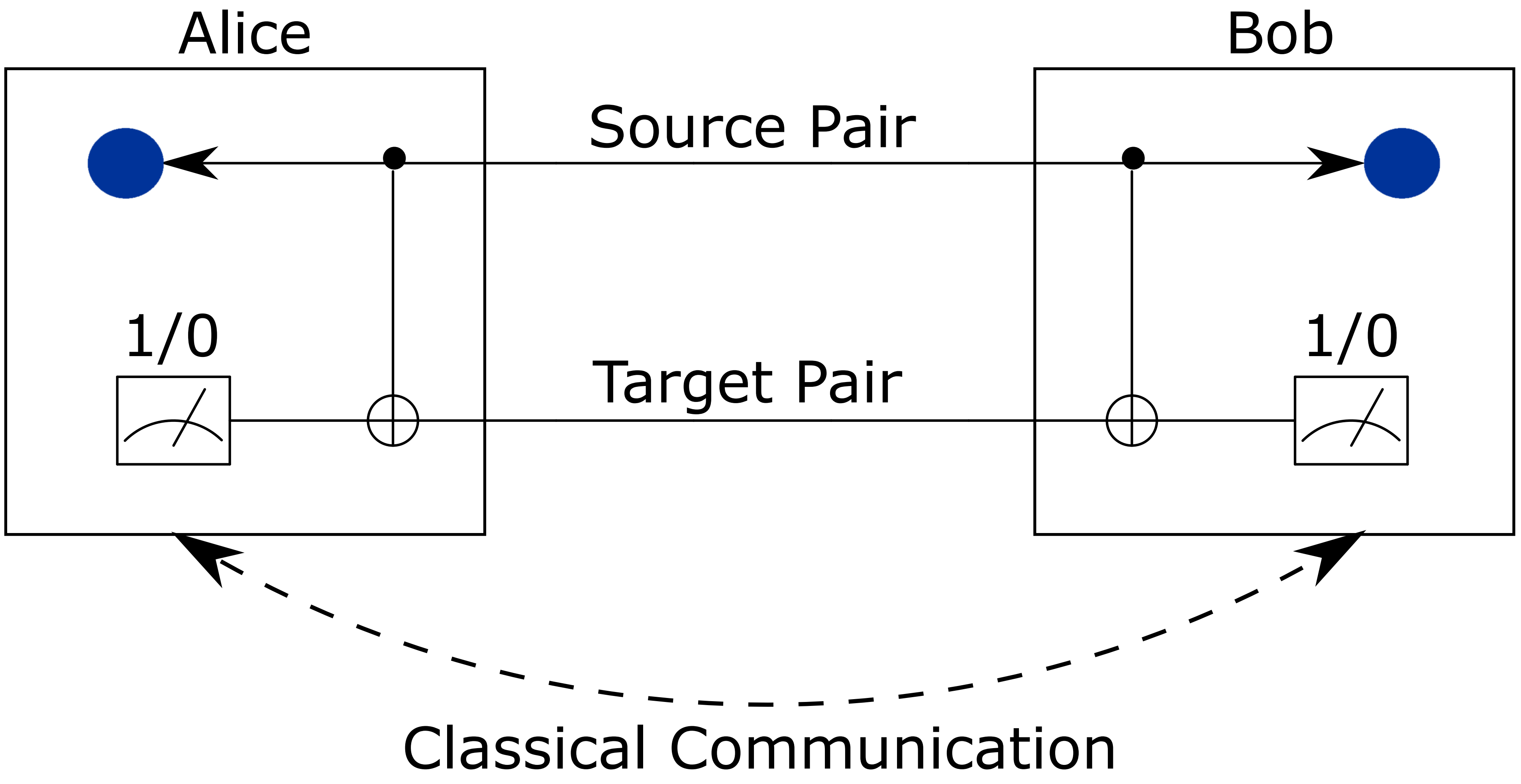
Entanglement purification



Probabilistically convert multiple noisy entangled pairs into single strongly entangled pair!

Phys. Rev. Lett. **76**, 722 (1996)

QoS metric

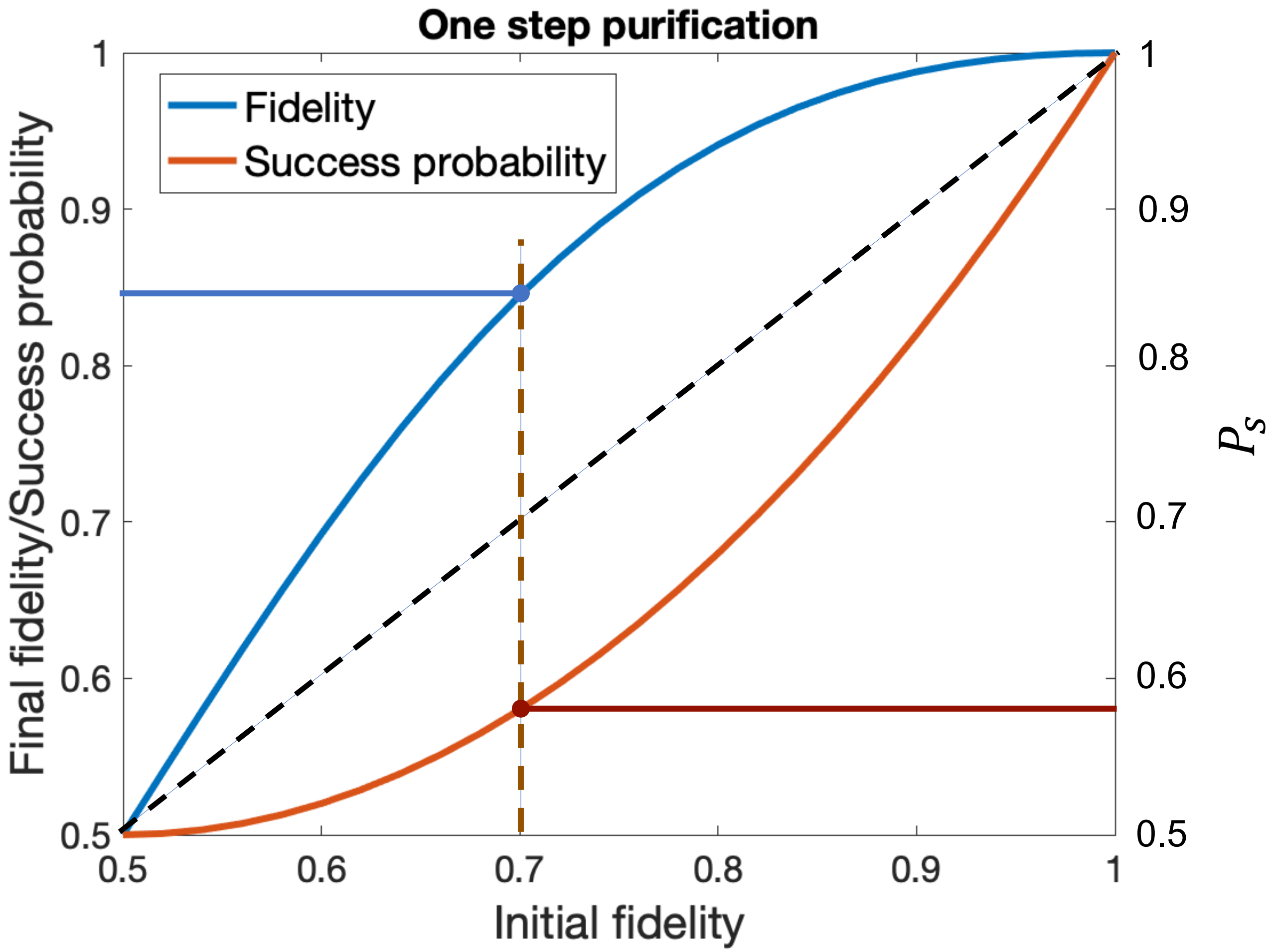
Fidelity: measure of closeness of entanglement to perfection

0 1

complete fidelity

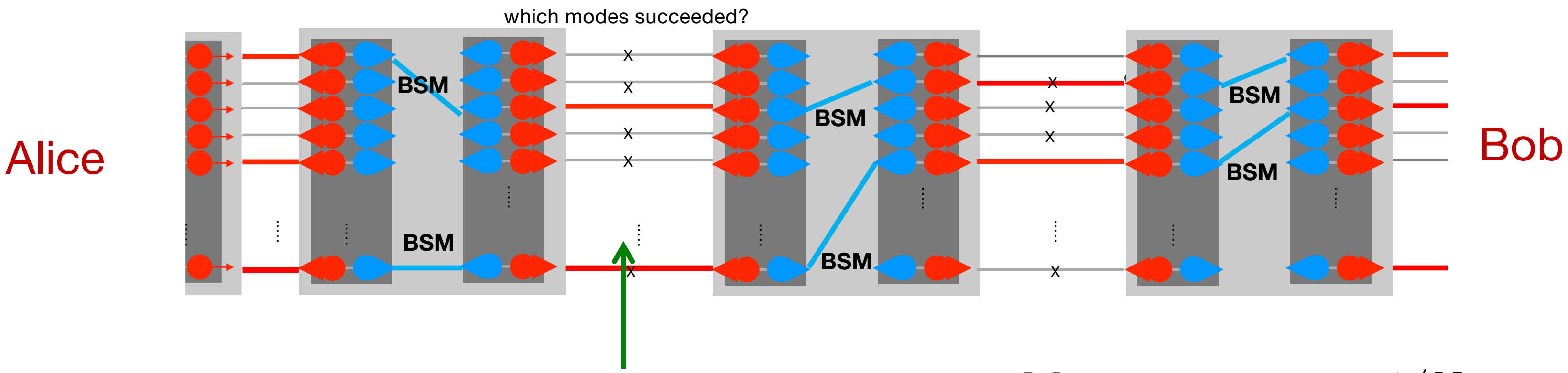
decreasing fidelity

Purification step succeeds with probability P_s



Back to linear repeater network

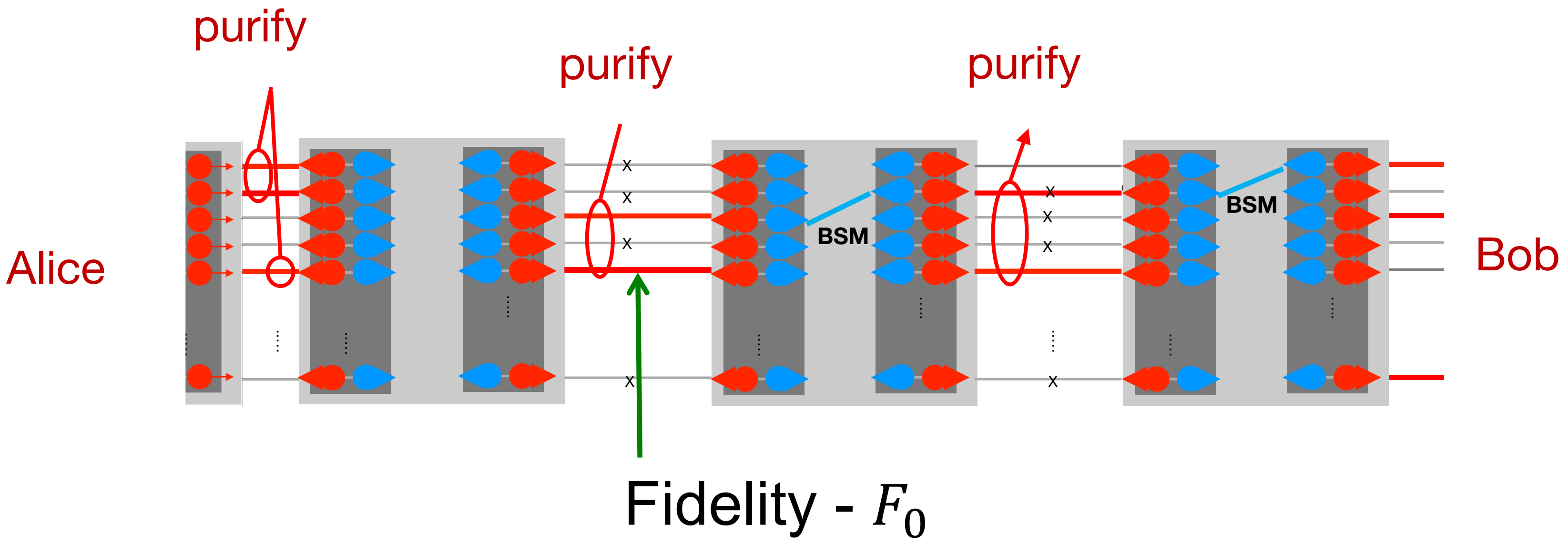
- Links consists of modes
  - spatial (frequencies, polarizations)
  - temporal



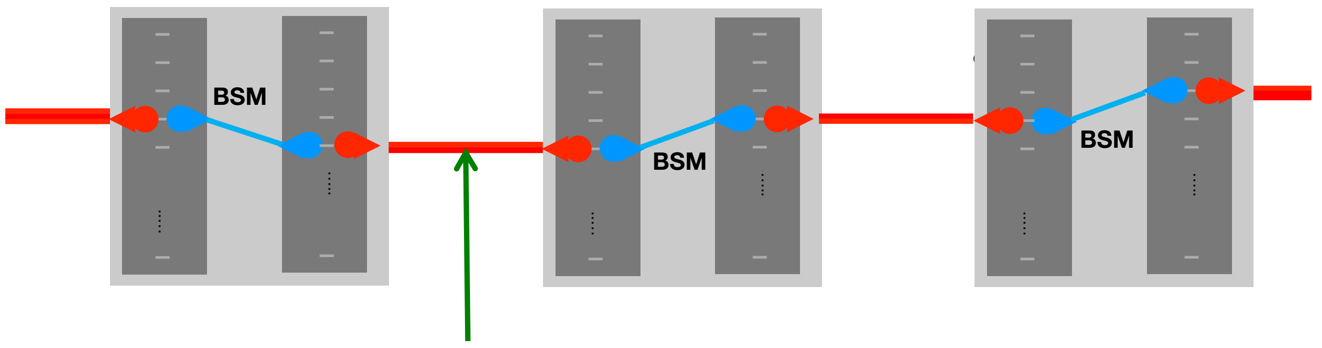
$$p = 1 - (1 - p_0)^M \qquad p_0 = c\,\eta^{1/N}$$

- Increases link success probability p
- Provides opportunity for purification

Purification



Purification



Final fidelity - $F' > F_0$

- Determine when and how much to purify
- Whether to purify across single or multiple links
- Possibly with minimum e2e fidelity constraint

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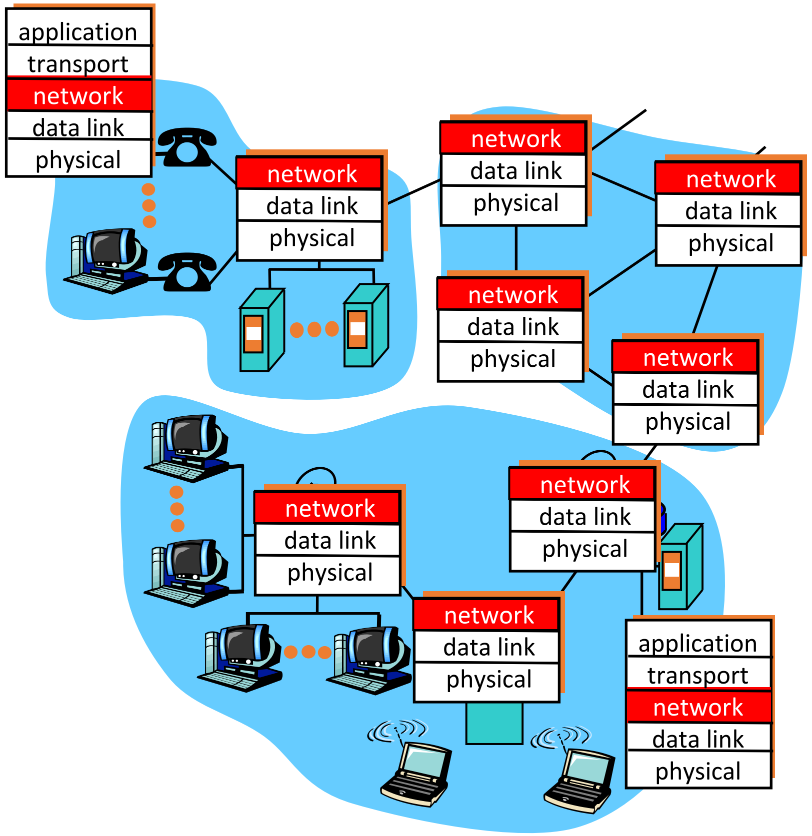
Network layer functions

- Transport packet from sending to receiving hosts
- Network layer protocols in *every* host, router

Three important functions:

- *Path selection:* route taken by packets from source to destination (routing algorithms)
- *Switching:* move packets from router's input to appropriate router output
- *Call setup:* some network architectures require router call setup along path before data flows



Network service model

CRUCIAL question!

Q: What *service model* for "channel" transporting packets from sender to receiver?

- guaranteed bandwidth?
- preservation of inter-packet timing (no jitter)?
- loss-free delivery?
- in-order delivery?
- congestion feedback to sender?

service abstraction

The most important abstraction provided by network layer:

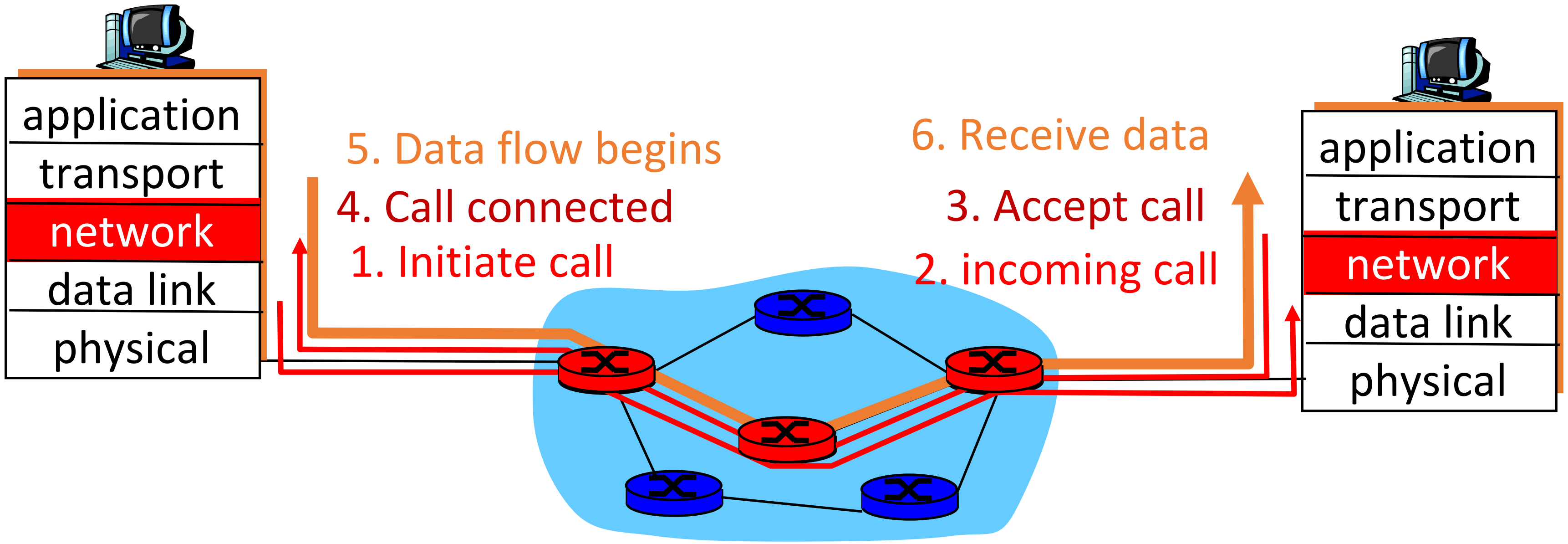
virtual circuit or datagram?

Virtual circuits

"source-to-dest path behaves like telephone circuit"

- performance-wise
- network actions along source-to-dest path

- Call setup, teardown for each call *before* data can flow
- Each packet carries VC identifier (not destination host ID)
- *Every* router on source-dest path maintains "state" for each passing connection
  - transport-layer connection only involved two end systems
- Link, router resources (bandwidth, buffers) may be *allocated* to VC
  - to get circuit-like performance


application
transport
network
data link
physical
5. Data flow begins
4. Call connected
1. Initiate call
6. Receive data
3. Accept call
2. incoming call
application
transport
network
data link
physical

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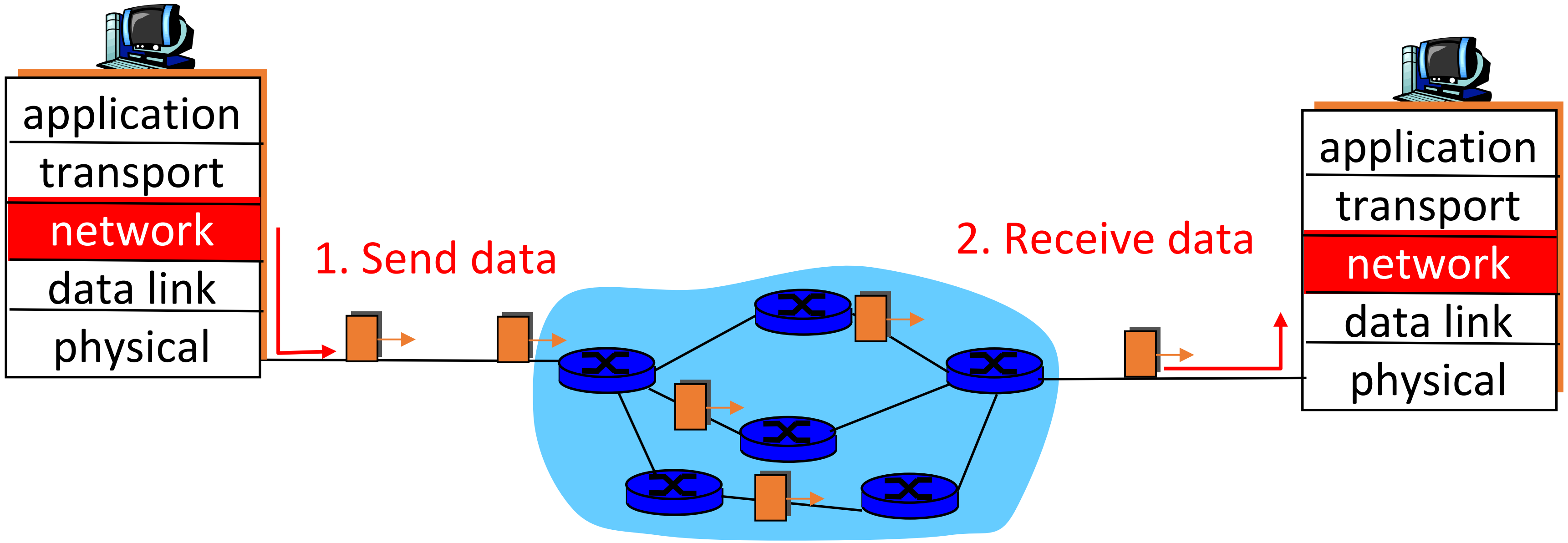
Datagram network: The Internet model

- No call setup at network layer
- Routers: no state about end-to-end connections
  - no network-level concept of "connection"
- Packets typically routed using destination host ID
  - packets between same source-dest pair may take different paths



Quantum network service model

CRUCIAL question!

Q: What *service model* for "quantum channel" between end nodes?

service abstraction

- guaranteed rate?
- latency guarantee?
- minimum fidelity guarantee?

The most important abstraction provided by network layer:

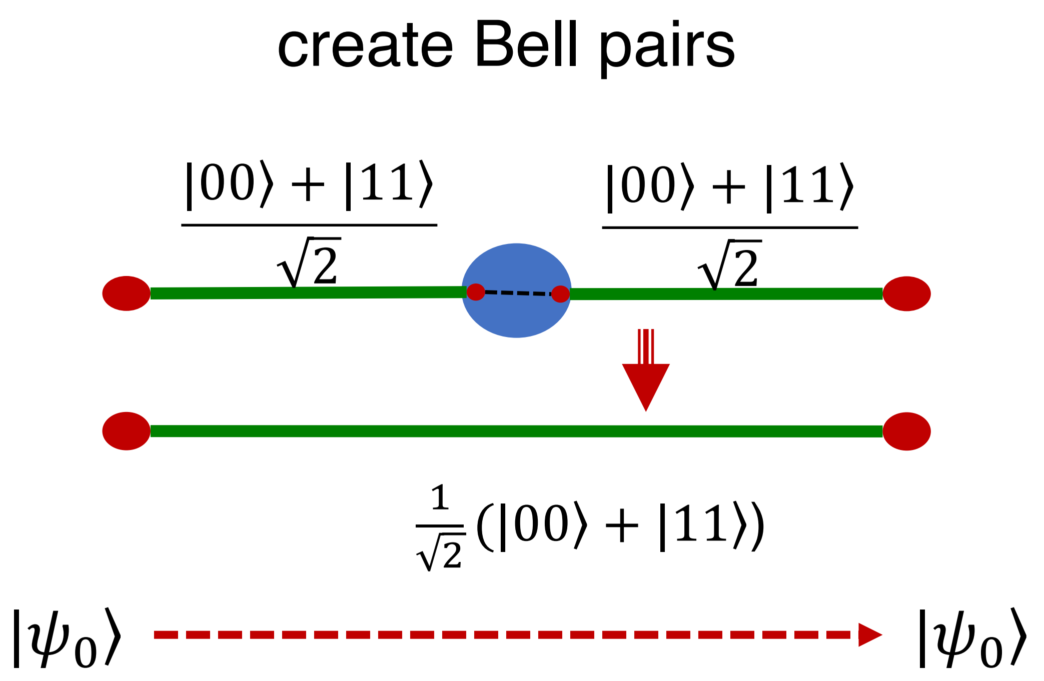
entanglement generation or quantum information transmission

? ? ?

Entanglement distribution

(Two-way network architecture)

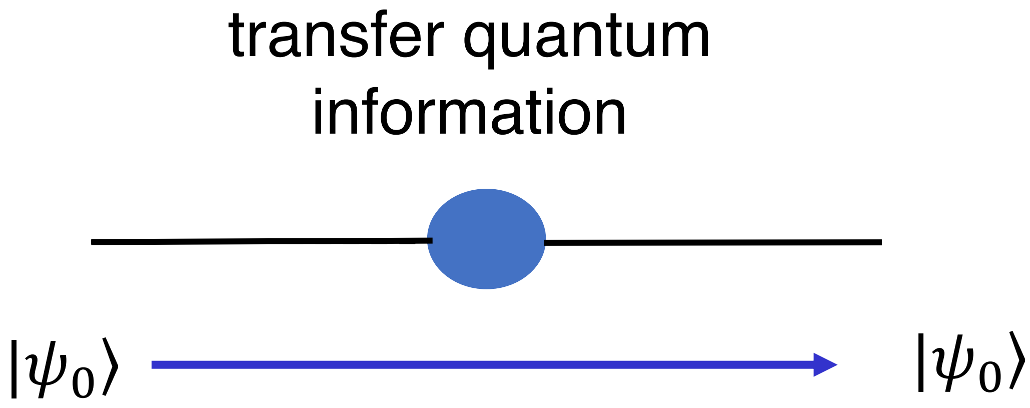
- Creation/distribution of Bell pairs (entanglement)
- Use teleportation to transfer quantum information
- Relies heavily on purification to handle noise
- Requires exchange of classical information for correction



Quantum information transfer

(One-way network architecture)

- Transfer quantum information directly
- Note resemblance to classical network
- Relies heavily on Quantum Error Correction (QEC)
- Does not require exchange of classical info

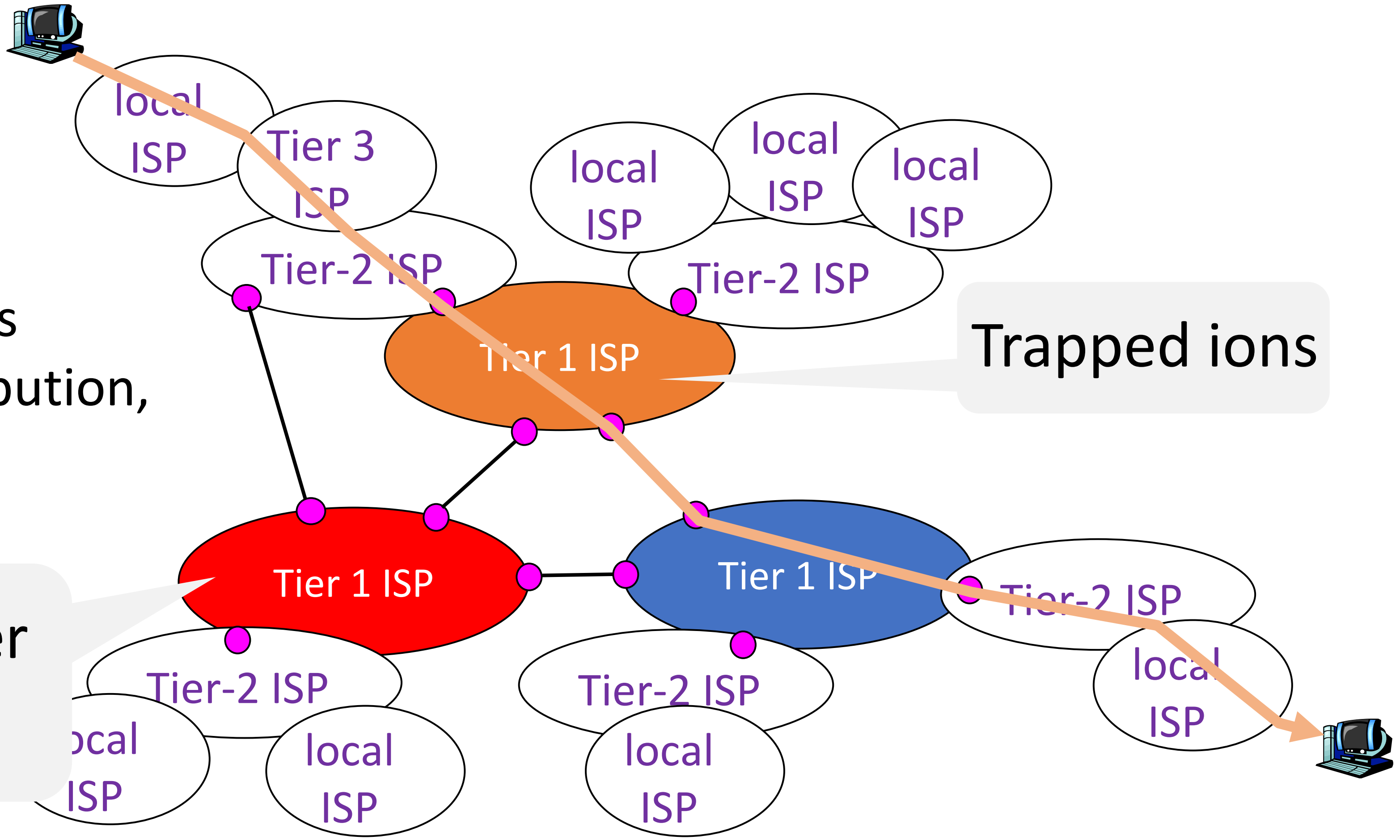


Note: services are interchangeable

Quantum Internet

- Quantum information can pass through many networks!
- e2e entanglement over many networks

Challenge:
some ISPs distributes entanglement distribution, others transmits QI



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One way vs. Two way

Two way

Pros:

- Purification simpler than QEC
- Bell pairs fungible ⇒
  - high rates
  - pre-shared entanglement
- Tolerates noisy gates

Cons:

- Increased latency due to classical comms
- High memory requirement

One way

Pros:

- No classical comms ⇒ low latency
- Low memory requirement

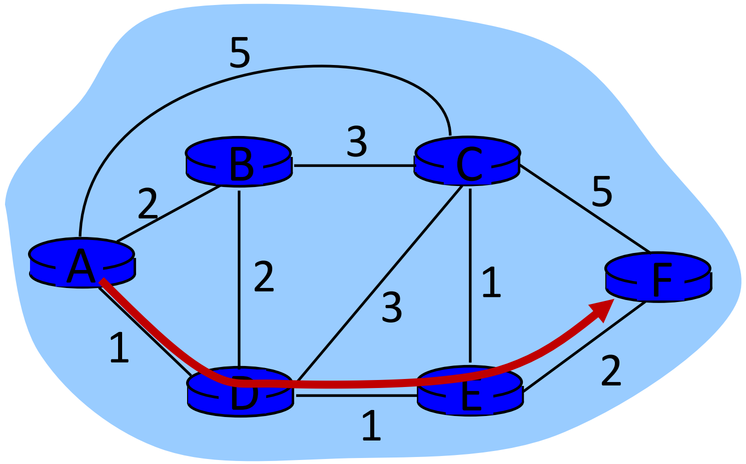
Cons:

- QEC very challenging, requires high quality gates
  - 100 physical qubits per logical qubit?
- Requires high quality gates

Classical routing

Routing protocol

Goal: determine "good" path (sequence of routers) thru network from source to dest.



Graph abstraction for routing algorithms:

- graph nodes are routers
- graph edges are physical links
  - link cost: delay, $ cost, or congestion level

"good" path:

- typically means minimum cost path
- other def's possible
- Dijkstra algorithm

Routing algorithm classification

Q: global or decentralized information?

global:

- central controller has complete topology, link cost info

Decentralized:

- router knows physically-connected neighbors, link costs to neighbors
- iterative process of computation, exchange of info with neighbors
- "distance vector" algorithms

Q: static or dynamic?

static:

- routes change slowly over time

Dynamic:

- routes change more quickly
  - periodic update
  - in response to link cost changes

Current approach

- (Logical) central controller with complete topology, link cost info
- Includes policy constraints
  - e.g., party A cannot use link set $\mathcal{L}$
- Calculation of backup paths
- Diversity for load balancing

Quantum routing

Static algorithms:

- shortest paths with link costs:
  - link entanglement rate, $1/R_l$
  - link fidelity, F_l
  - and others

Dynamic algorithms:

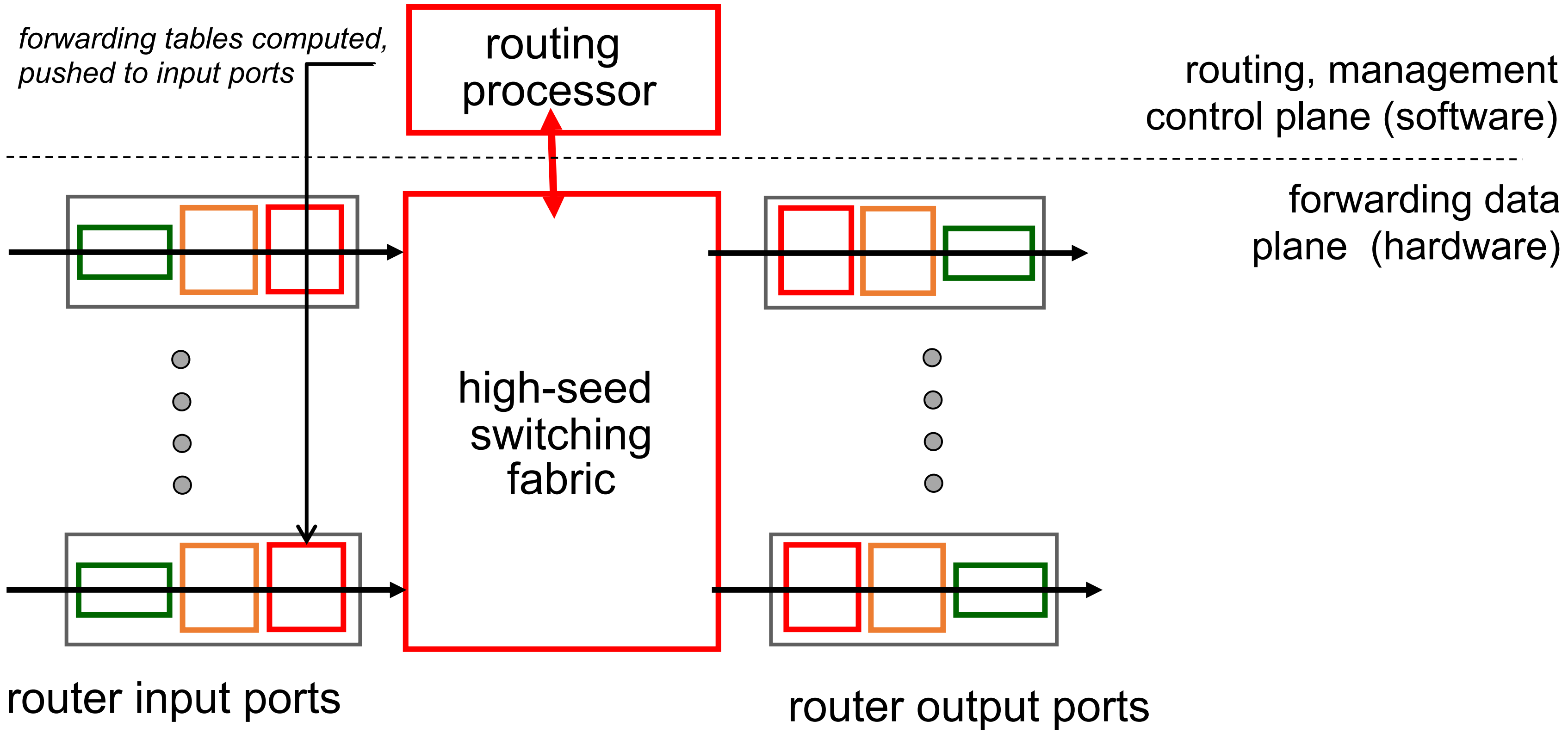
- each node chooses neighbors to connect based on local state information

Classical routers & quantum switches

Classical router architecture overview

two key router functions:

- run routing algorithms/protocol
- *forwarding* packets from incoming to outgoing link



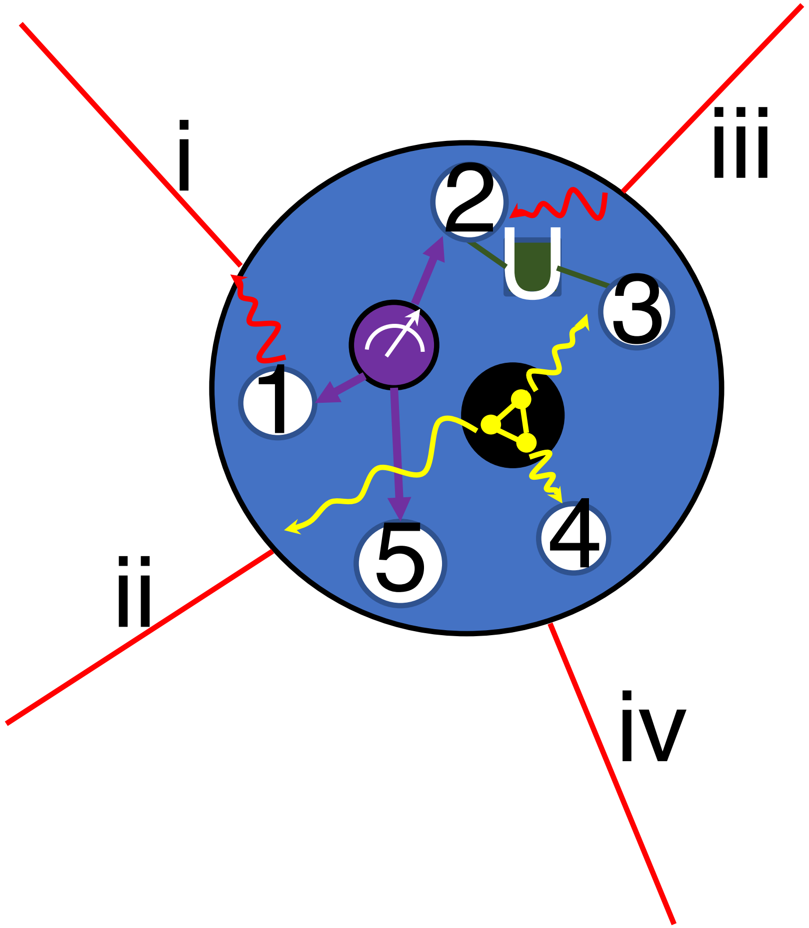
Challenges

- capacity of router?

- scheduling policies that achieve capacity? that reduce switching fabric complexity?
  - matching algorithms
  - max weight policies
  - lightweight randomized algorithms

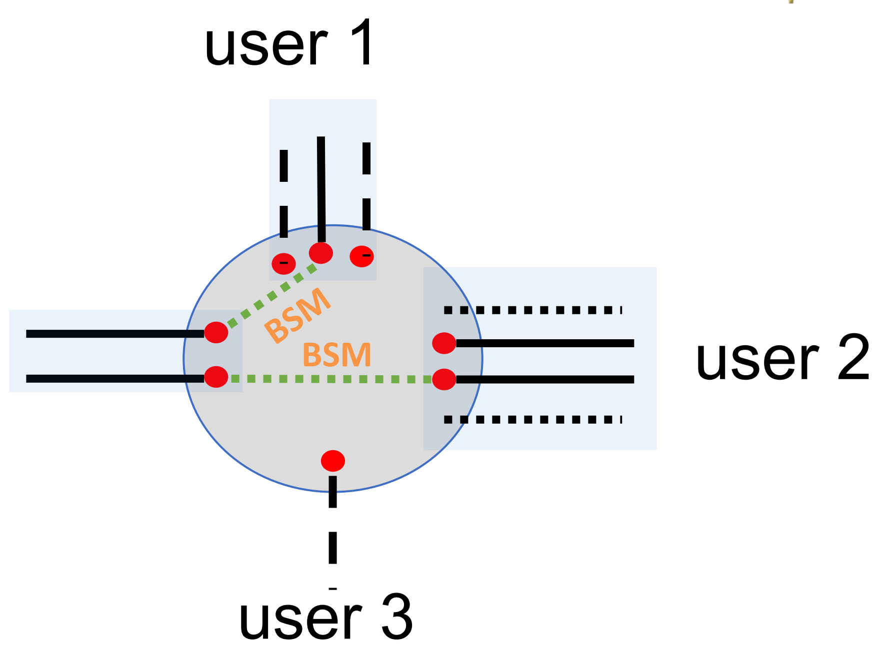
Quantum switch

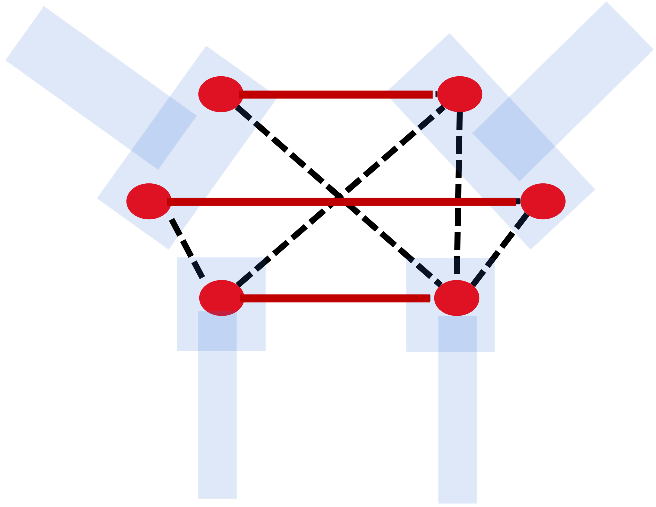
- Quantum memories: loading and readout
- Multi-qubit quantum measurements
- Quantum logic across qubits held in QMs
- Multi-photon entanglement sources
- Classical computing and communications



Quantum switch

- User pairs generate requests for Bell pairs
- Phase 1: links randomly generate Bell pairs
- Phase 2: given outstanding requests, switch selects Bell pairs to measure
  - equivalent to selecting eligible matching in a graph among memories
- Outcomes of BSM matchings form set of end-to-end entanglements between pairs of end nodes





Challenges

- switch design, switching fabric
  - teleportation fabric?
- network capacity, network resource allocation
  - global vs local vs no state information
  - timescale of state information
- memory decoherence, gate errors?
- quality of information – fidelity
  - fidelity degrades over time ⇒ last in first out (LIFO), deadline scheduling?

⇒ (virtual) circuit switching?

Summary

- entanglement distribution service very different from quantum information transfer service

- quantum networking introduces new problems

  … and old problems with new wrinkles

  - resource allocation, path selection, switch & entanglement scheduling
  - delivery of QoS in very noisy environment

- research on Q-networks in its infancy with many exciting problems!

Questions?

Capacity and Resource Allocation

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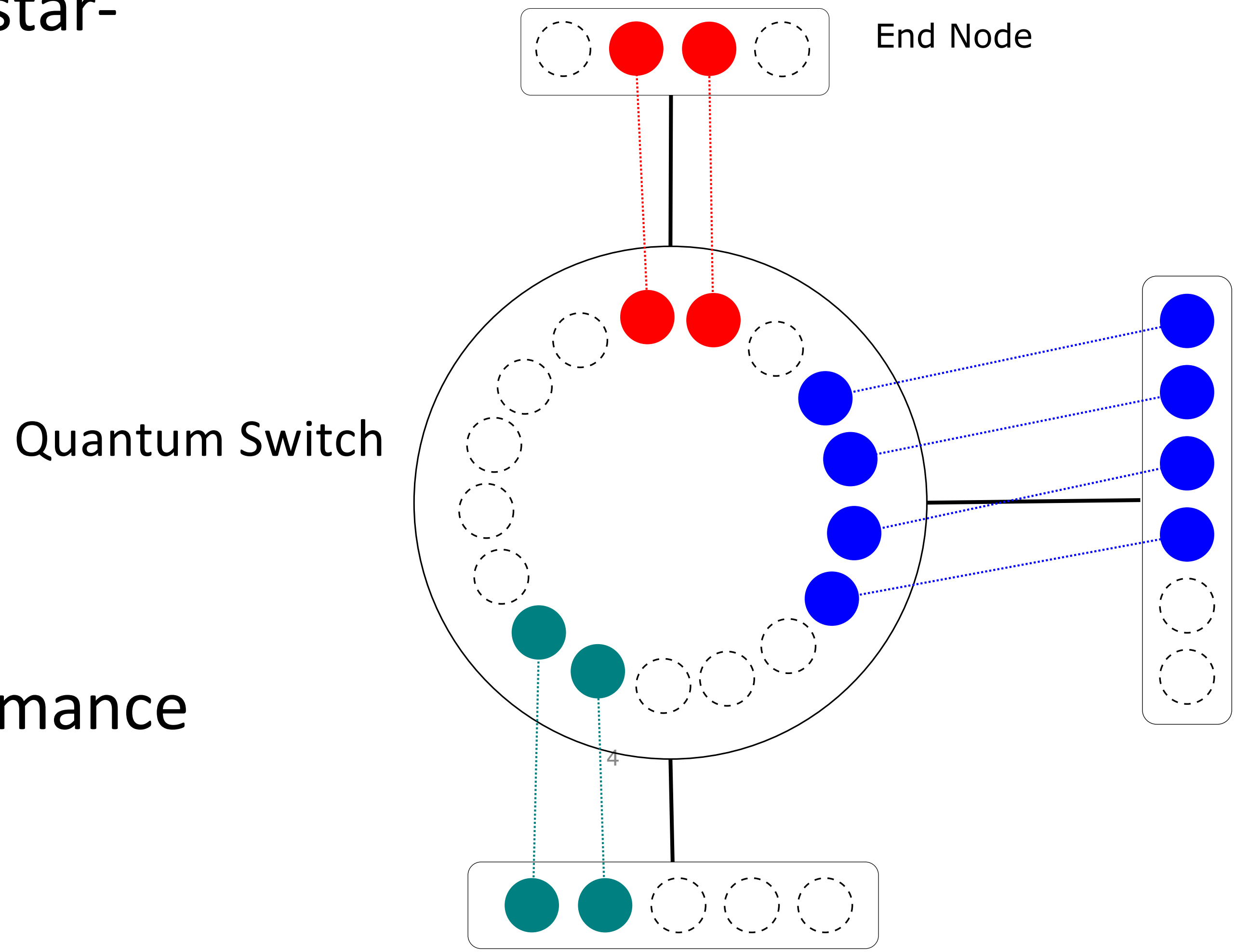
Outline

Network capacity	Stability analysis
Resource allocation for achieving capacity	Stability analysis
Scheduling to mitigate against memory noise	Markov processes
Path selection	Percolation theory
Flow & swap optimization	Linear programming, optimization theory

Quantum Switch

- Quantum switch: center node of a star-shaped network
  - end nodes
  - quantum channels

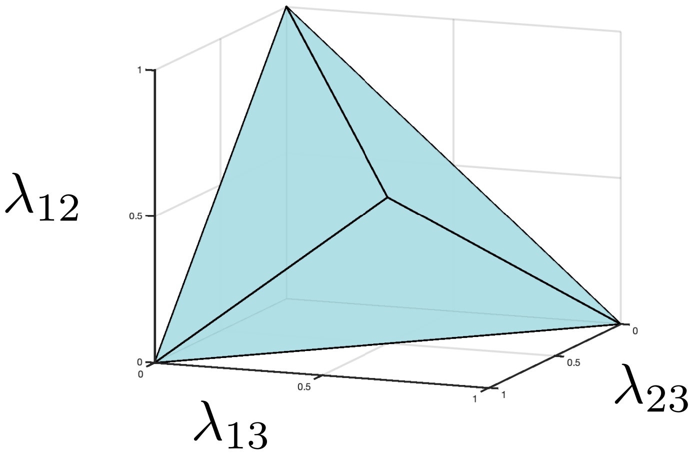


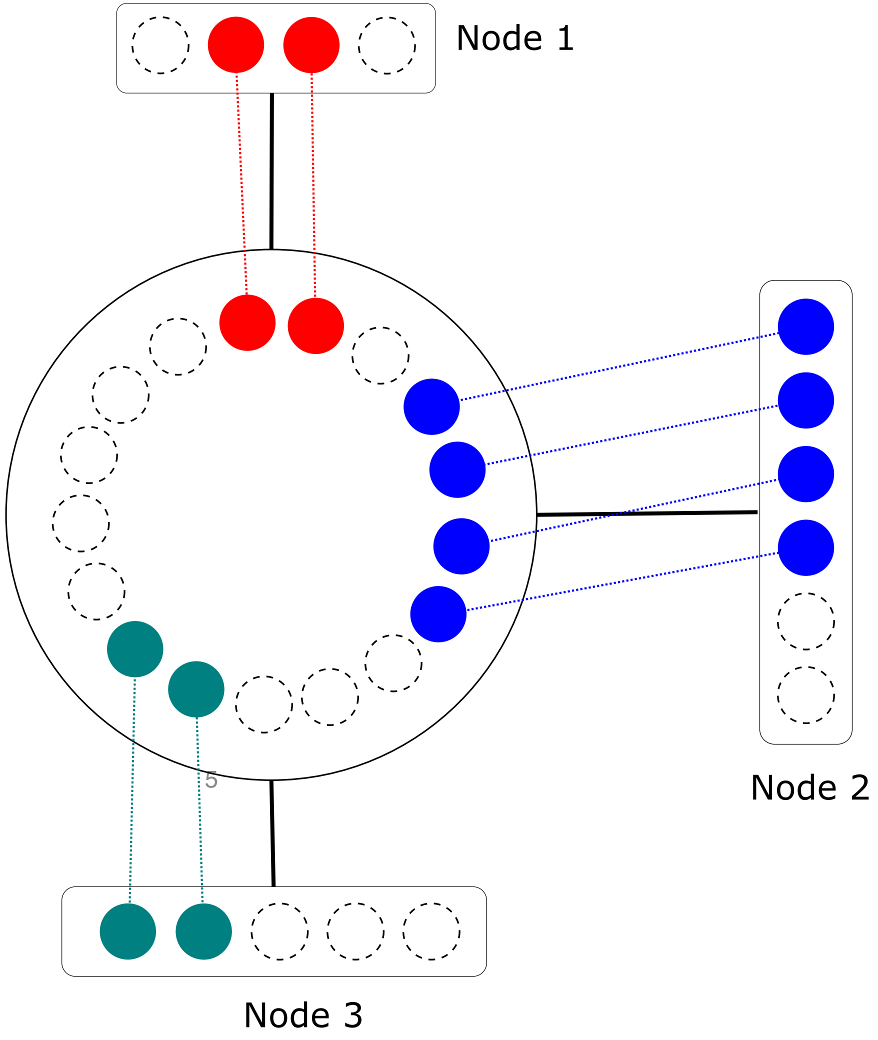
- How do we achieve the best performance with multiple source-sink pairs?

- How to quantify the performance?

Capacity Region

- Entanglement requests randomly arrive at switch with infinite memory
- Requests have rates: $\lambda_{12}, \lambda_{13}, \lambda_{23}, \ldots$
- Stability: quantum switch is stable if request delays are finite
- Capacity region: set of request rate vectors such that switch can be stabilized

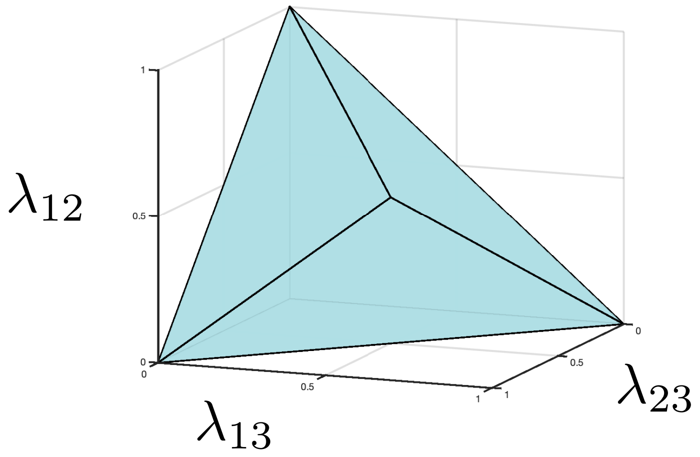


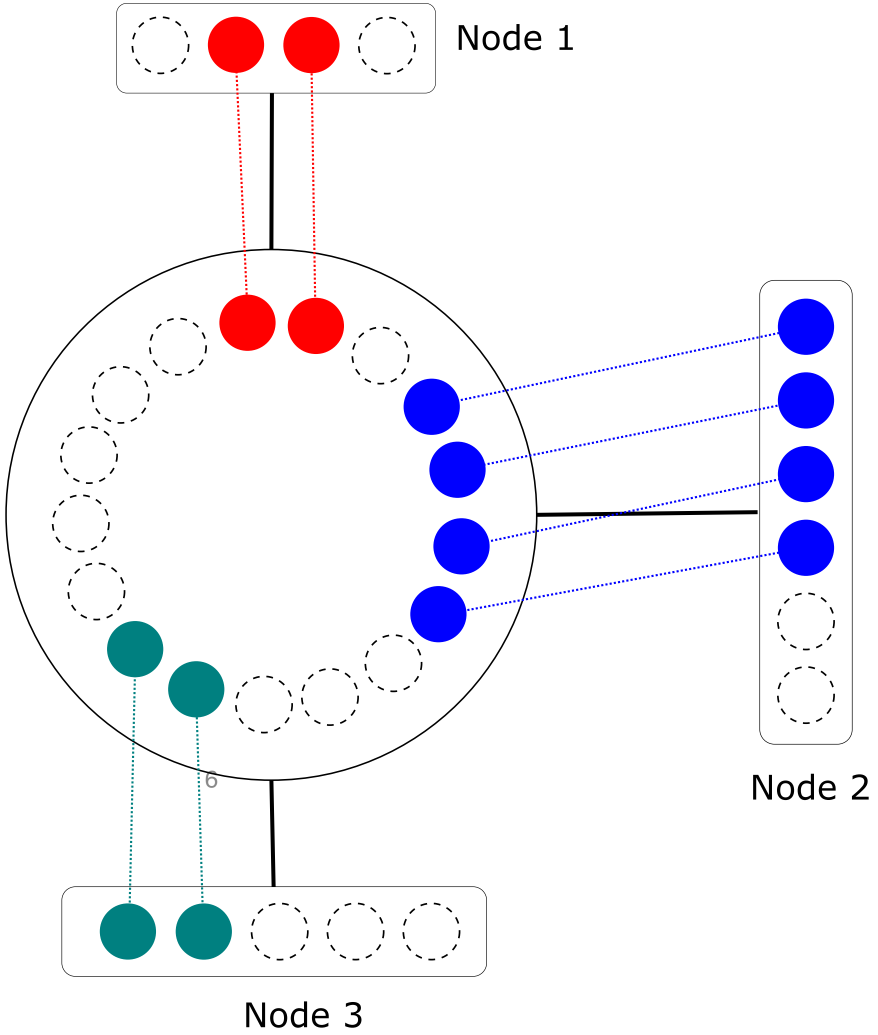


Capacity Region

Two sides of story:

- unstable outside region
- design scheduling algorithms that stabilize switch inside region (who to swap)





System Model

Slotted time:

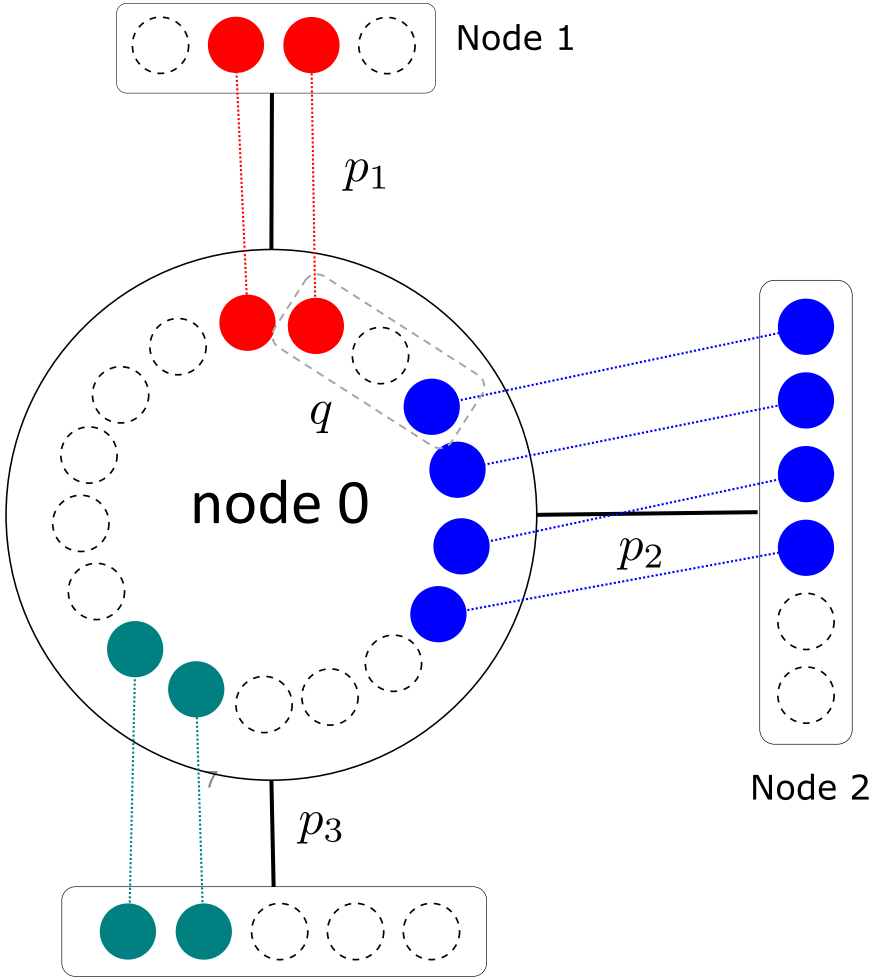
Entanglement generation: entanglement $|\Psi_{0k}\rangle$ successfully generated with probability p_k

Entanglement swapping: entanglement $|\Psi_{ij}\rangle$ created with probability q by consuming $|\Psi_{0i}\rangle$ and $|\Psi_{0j}\rangle$

Entanglement requests: $\{A_{ij}(t): t \geq 0\}$ randomly arrive at switch, arrival rates $\{\lambda_{ij}\}$

$\lambda_{ij} < 1$, interpret as probability Bell pair requested in slot

Perfect memory, infinite memory at switch and end-nodes



Stability

Theorem: Capacity region is set of all vectors $\{\lambda_{i,j}\}$ for which

$$\sum_i \lambda_{ij}/q \le p_j, \qquad \forall j$$

Intuition:

- expected number of swap attempts per successful swap for each (i,j) request – $1/q$
- after a long time T, roughly speaking $\lambda_{i,j}T/q$ swap operations each consuming one of each $|\Psi_{0i}\rangle$ and $|\Psi_{0j}\rangle$
- requires $\sum_i \lambda_{ij}T/q \le p_j T$ pairs of $|\Psi_{0j}\rangle$, $\forall j$

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Resource allocation

Stationary resource allocation

- label each generated $|\Psi_{0i}\rangle$ as (*i*, *j*) with probability $f_{ij} = \frac{\lambda_{ij}}{\sum_j \lambda_{ij}} \geq \lambda_{ij}$

  (i, j) is equivalent to (j, i)
- swap $|\Psi_{0i}\rangle$ and $|\Psi_{0j}\rangle$ if both labelled (i, j)

Why it works:

- after long time T, roughly speaking $p_i T$ pairs of $|\Psi_{0i}\rangle$ generated
- $p_i T f_{ij}/p_i \geq \lambda_{ij} T$ pairs of $|\Psi_{0i}\rangle$ labelled as (i, j)
- similar number of $|\Psi_{0j}\rangle$ labelled as (i, j)
- swapping yields

$$q f_{ij} T \geq \lambda_{ij} T$$

Resource allocation

Stationary resource allocation

- label each generated $|\Psi_{0i}\rangle$ as (*i, j*) with probability $f_{ij} = \frac{\lambda_{ij}}{\sum_j \lambda_{ij}} \geq \lambda_{ij}$

  (i, j) is equivalent to (j, i)
- swap $|\Psi_{0i}\rangle$ and $|\Psi_{0j}\rangle$ if both labelled (*i, j*)

Proof that this algorithm is stable for any $\{\lambda_{ij}\}$ relies on Lyapunov stability theory

[details in arxiv.org/abs/2110.04116]

Resource Allocation: Remarks

Suppose $\{\lambda_{ij}\}$ "strictly" in capacity region;

then $Tf_{ij} > \lambda_{ij}T$ pairs of $|\Psi_{0i}\rangle$ labelled as (i,j)

Can store excess at end nodes to serve future requests

(preshared entanglement)

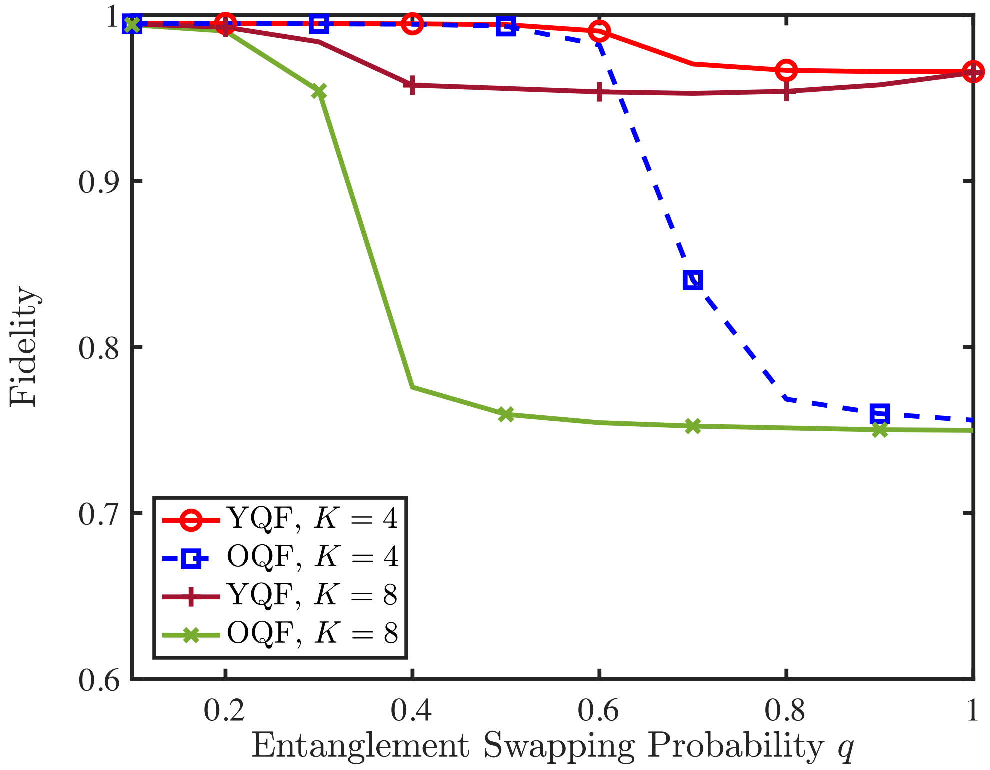
Provides zero latency service

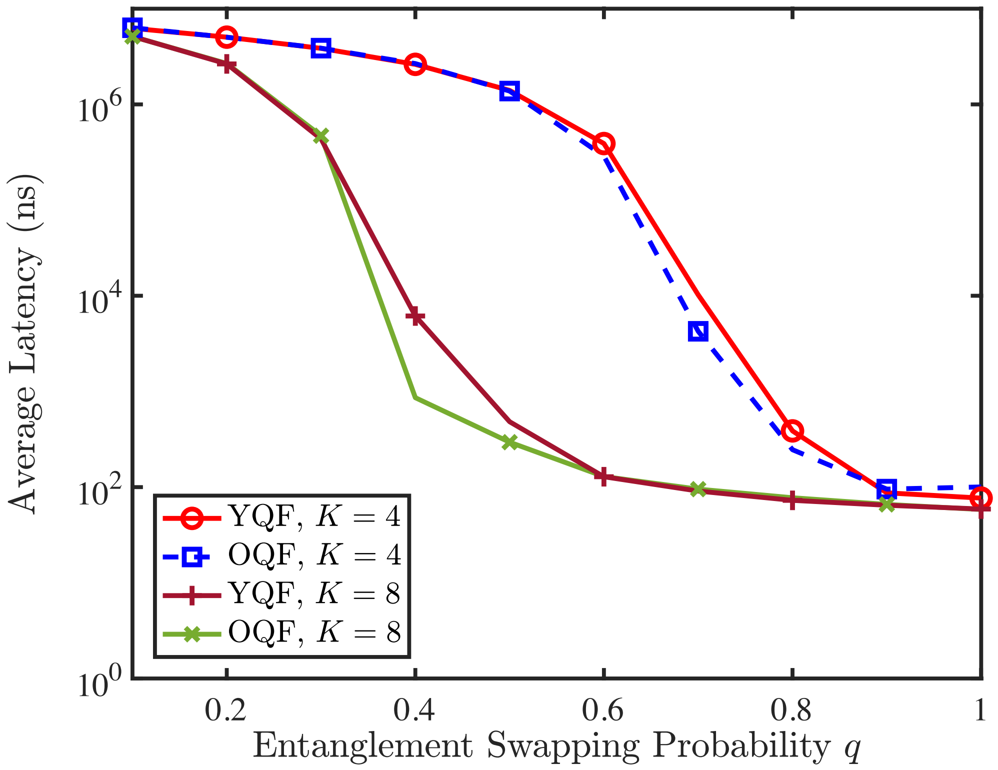
Simulation Setting

- Discrete event simulator: NetSquid
- Practical scenarios
  - decoherence in memories
  - finite number of memories
- Metrics:
  - average fidelity - F
  - average latency
- Prioritization
  - EPR pairs: Oldest-Qubit-First (OQF) and Youngest-Qubit-First (YQF)
  - entanglement requests: First-In-First-Out (FIFO)
- Discard qubits when fidelity is lower than a preset threshold

Entanglement Swapping Probability

- Fidelity, latency vs. entanglement swapping probability
- Fidelity, latency initially decreases with q, then remains constant
- Change in fidelity, latency occurs at $q = 0.33$ $(K = 8)$ &. $q = 0.67$ $(K = 4)$



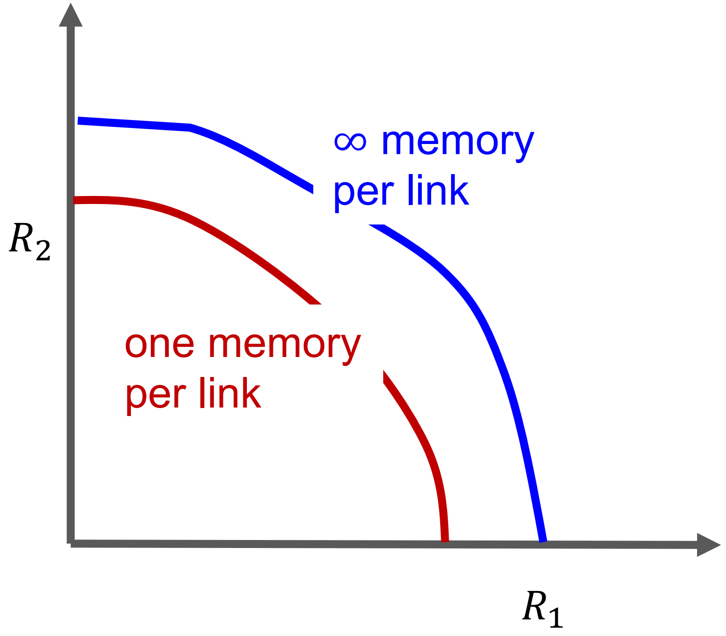


Extensions

Other extreme: qubit decoheres after one slot

Theorem: (*T. Vasantam, DT, SPIE 2022*)

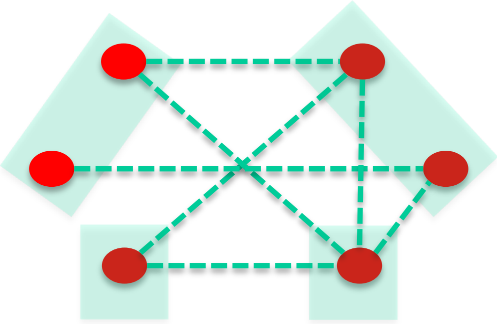
Capacity region characterization (more complicated than infinite memory)



Max-Weight policy stabilizes switch

matching π that maximizes

$$\sum_{ij} q\pi_{ij}\, Q_{ij}$$



Q_{ij} - number requests for i, j entanglement where link i, j entanglements exist

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Challenges

- Need to deal with noisy gates, memories
  - some initial results [Panigrahy, etal arxiv.2212.01463]
- Extend to network setting
  - characterization of capacity region probably straightforward
  - development of efficient scheduling algorithms – challenging
- Applications with different requirements

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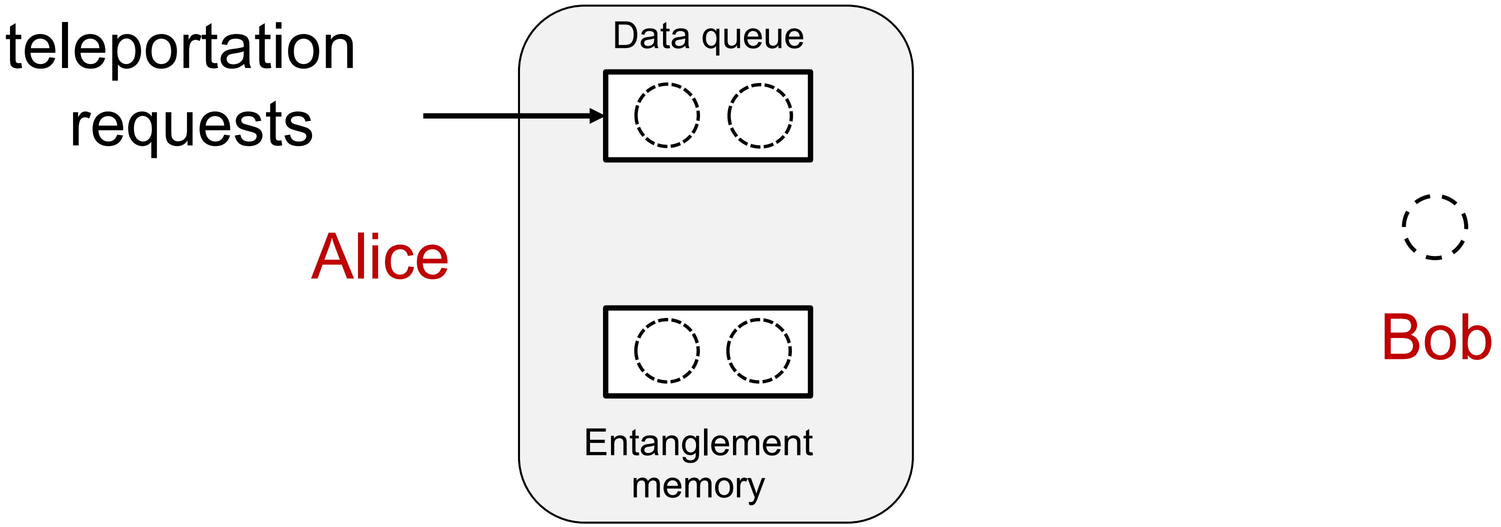
Modeling and reducing effect of memory noise

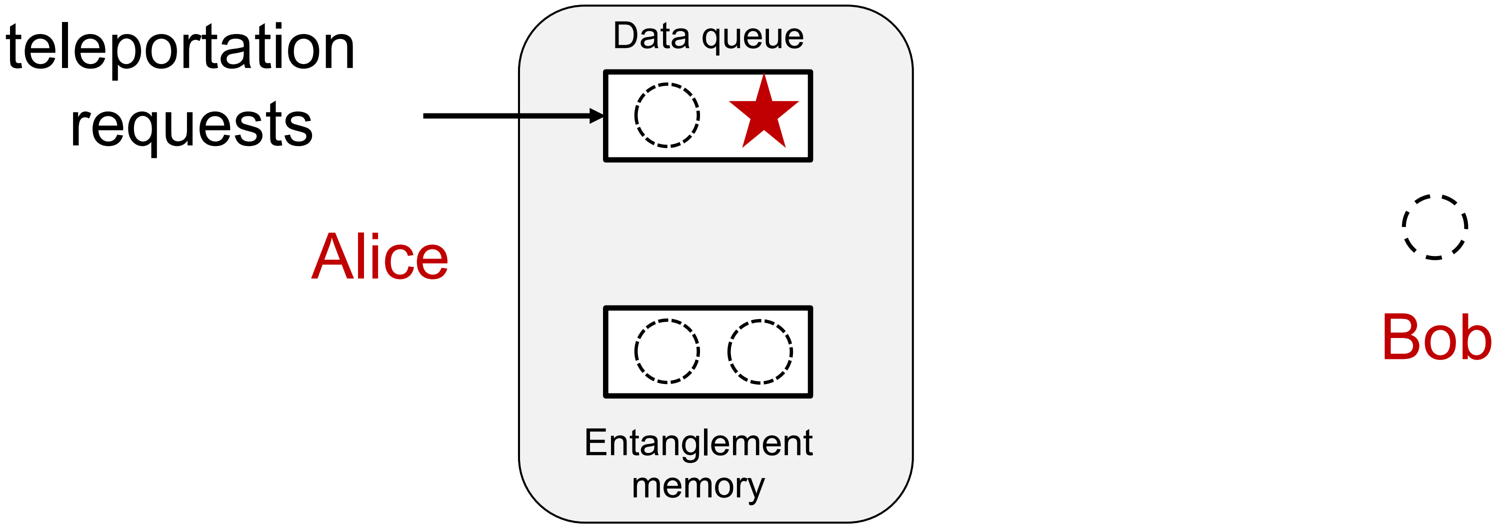
Scheduling teleportation

Quantum data transmission

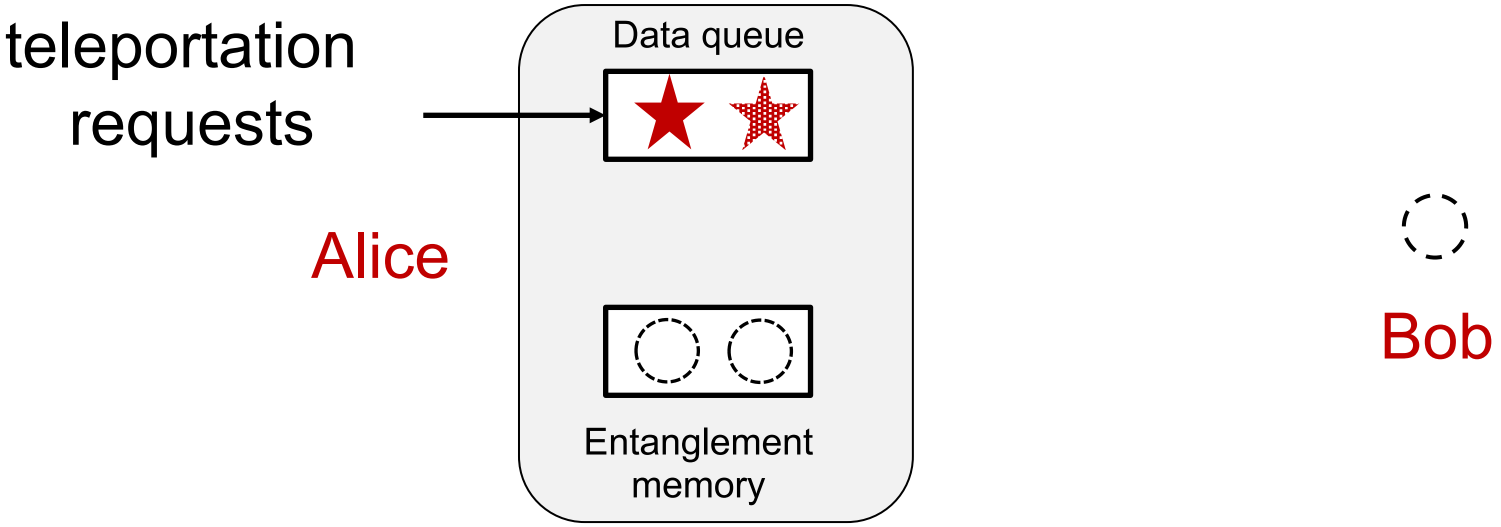
- data qubits, Bell pairs placed into memory
- served when paired



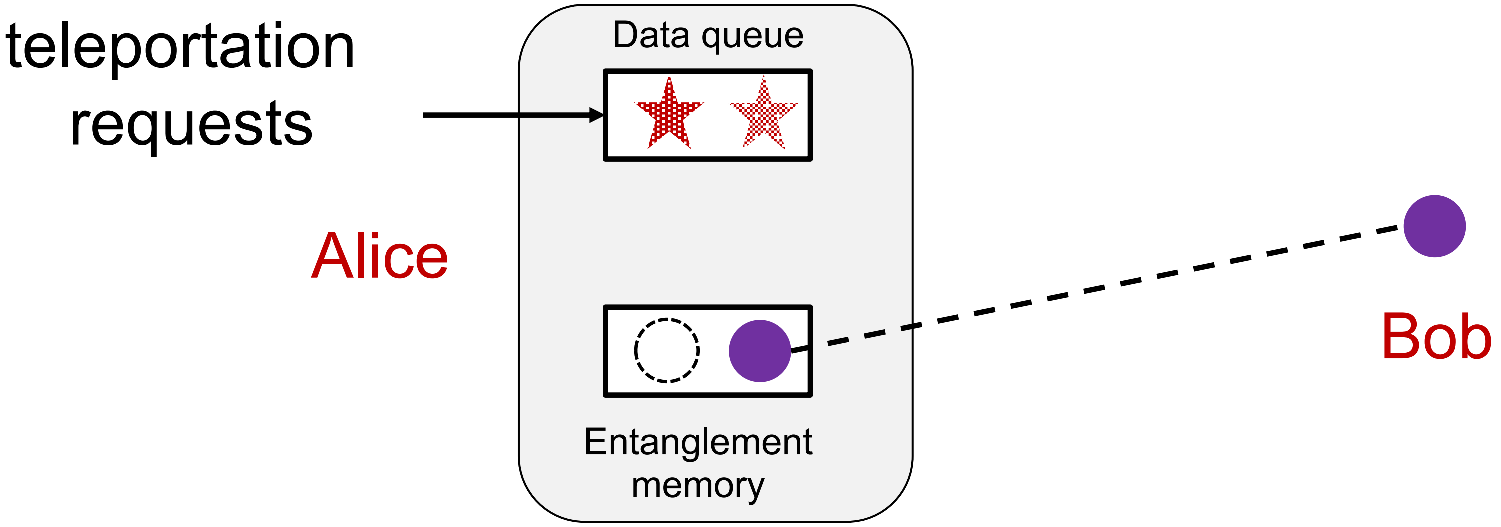
Scheduling teleportation



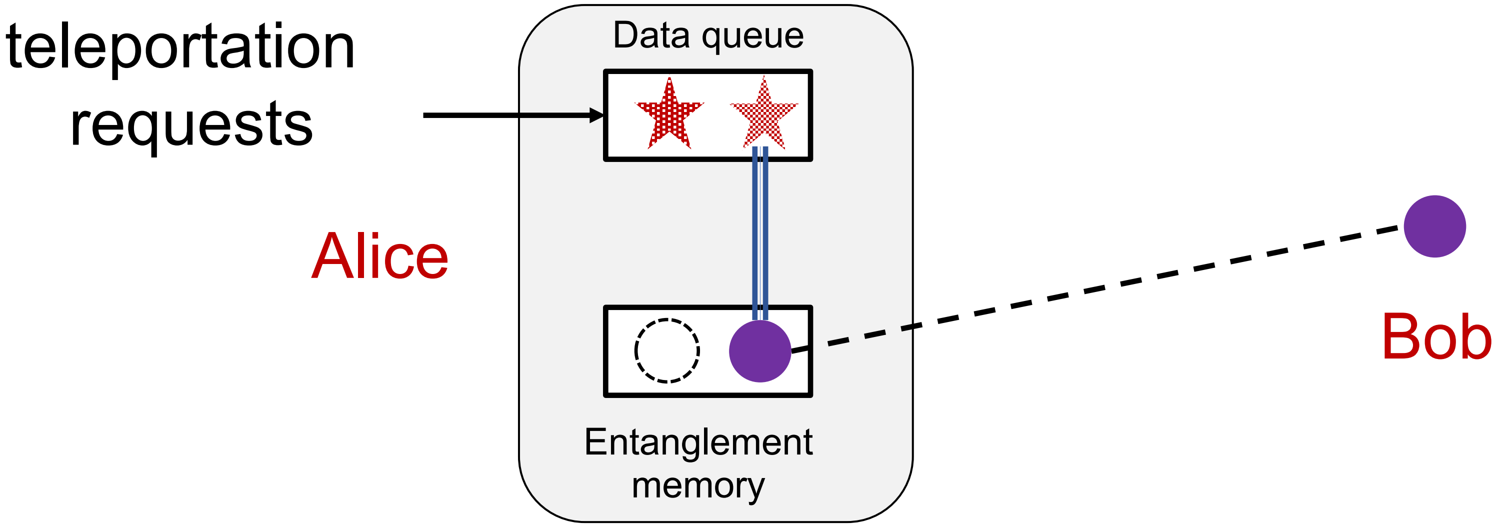
Scheduling teleportation



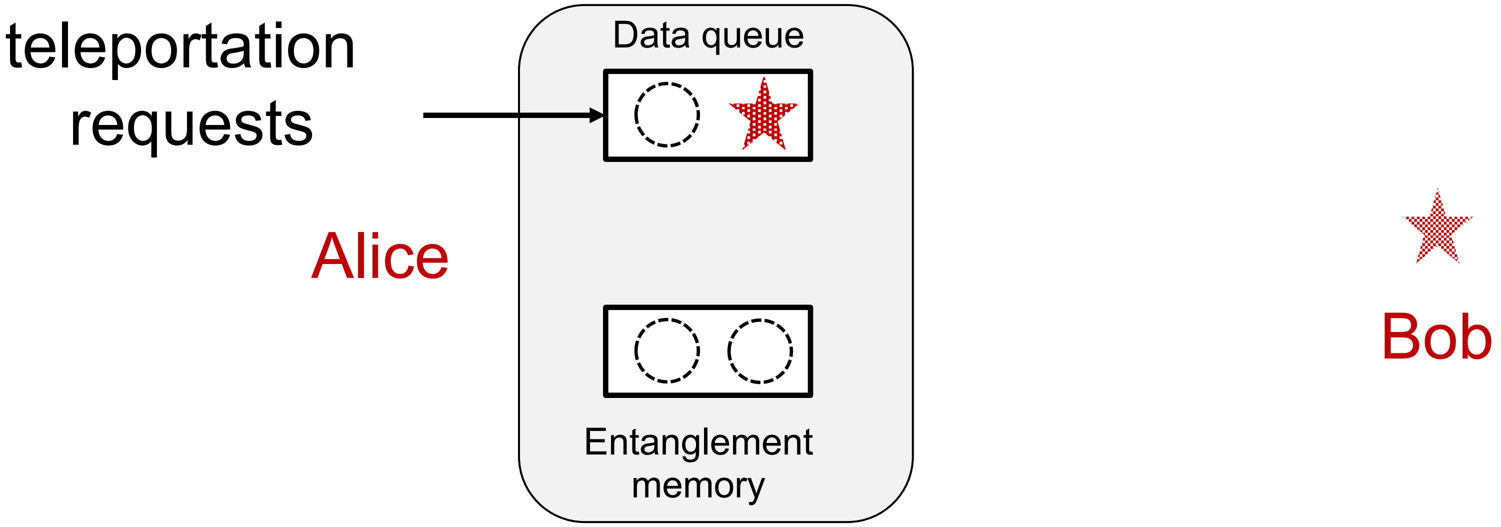
Scheduling teleportation



Scheduling teleportation



Scheduling teleportation



Resource management

How should Bell pairs and data qubits be scheduled?

- oldest qubit first (OQF)?
- youngest qubit first (YQF)?

How should buffer be managed?

- discard arrival?
- discard oldest entry (push out, PO)?

Modeling decoherence

- Fidelity most widely used measure of degradation due to noise
- Easy to compute for many (memory) noise models

- t – time quantum state spends in memory (single qubit, Bell pair)
- T_2- memory decoherence time
- $F(t)$ – fidelity of qubit spending time t in memory

$$F(t) = a + be^{-t/T_2}$$

where a, b, T_2 depend on noise model, quantum state, and technology,

$$a + b = 1$$

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Modeling decoherence

- T – time qubit spends in memory, $T \geq 0$
- $f_T(t)$ – probability density function for memory time T, $t \geq 0$.
- $F_T^*(s)$ – Laplace transform for T

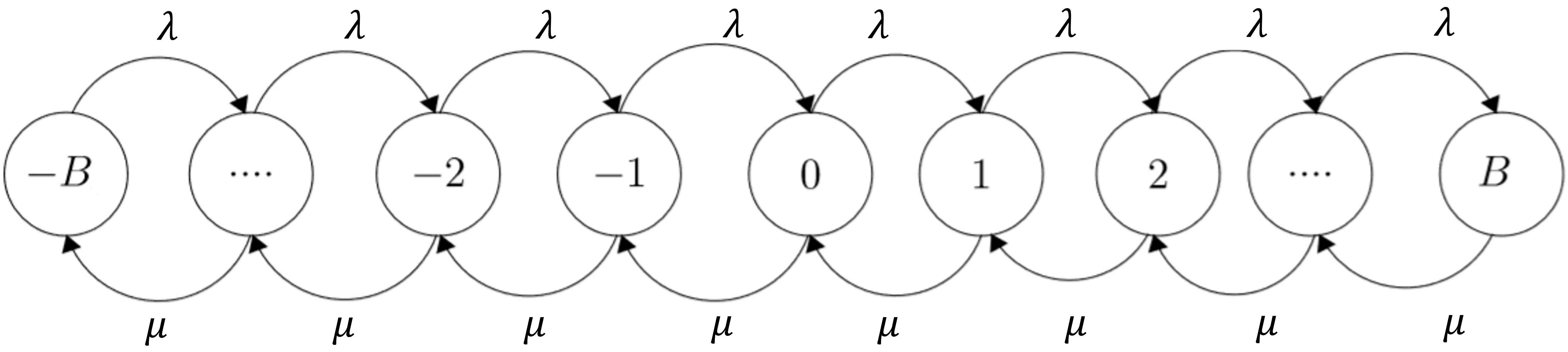
$$F_T^*(\mathrm{s}) = \mathrm{E}[e^{-sT}], \qquad s \geq 0$$

- F – fidelity
- Average fidelity:

$$E[F] = a + b\int_0^{\infty} f_T(t)e^{-t/T_2}dt$$
$$= a + b\,F_T^*(1/T_2)$$

Modelling resource management

- EPR pairs generated according to Poisson process, λ, cached in memory
- Teleportation requests generated according to Poisson process, μ, cached in memory
- Behavior described by continuous time Markov chain (CTMC)
- Memory size B



Putting it all together

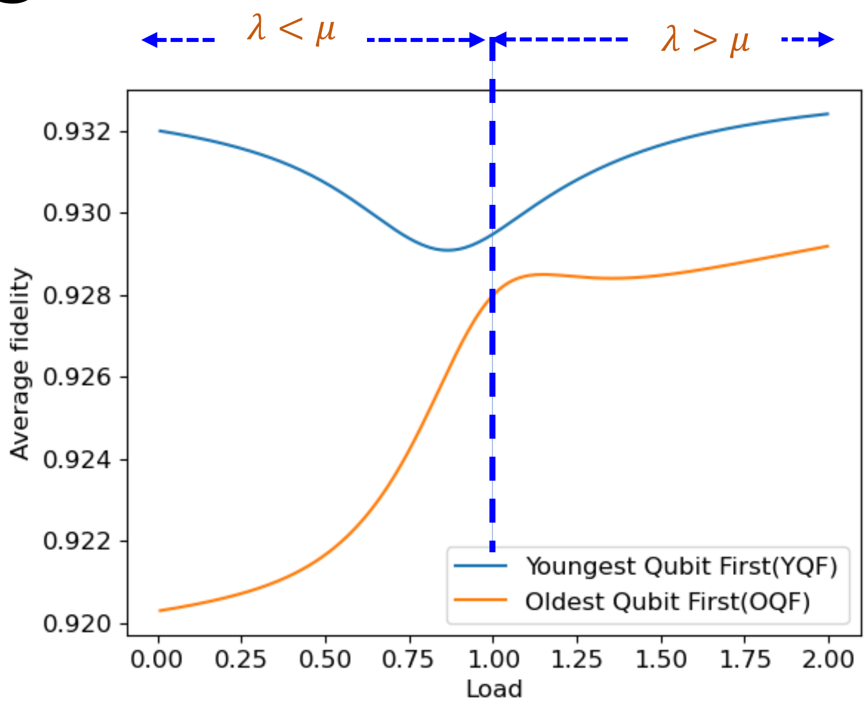
CTMC very easy to solve to obtain

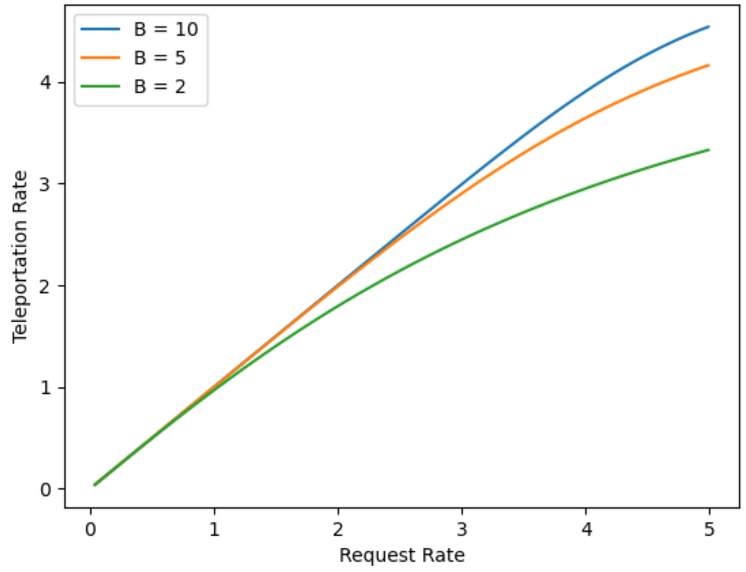
- distribution for number of occupied memories
- distribution and Laplace transform for time qubit resides in memory (R) prior to teleportation, $f_R(x), F_R^*(s)$

Results

- Poisson data generation - λ
- Poisson entanglement generation - μ
- load = λ/μ
- initial entanglement fidelity – 0.9; initial data fidelity 1
- fidelity decays exponentially in time
- memory size: 10
- policies YQF, OQF with pushout

YQF-PO provably optimal





Results

- Youngest qubit first with pushout maximizes entanglement rate, average fidelity

- Timeout schemes provide minimum fidelity guarantees

Challenges

- Does optimality of YQF extend to other settings?
  - linear repeater network
  - more general networks
- Can techniques be used to model network scenarios?
- Can models account for Bell pair generation, classical communications?

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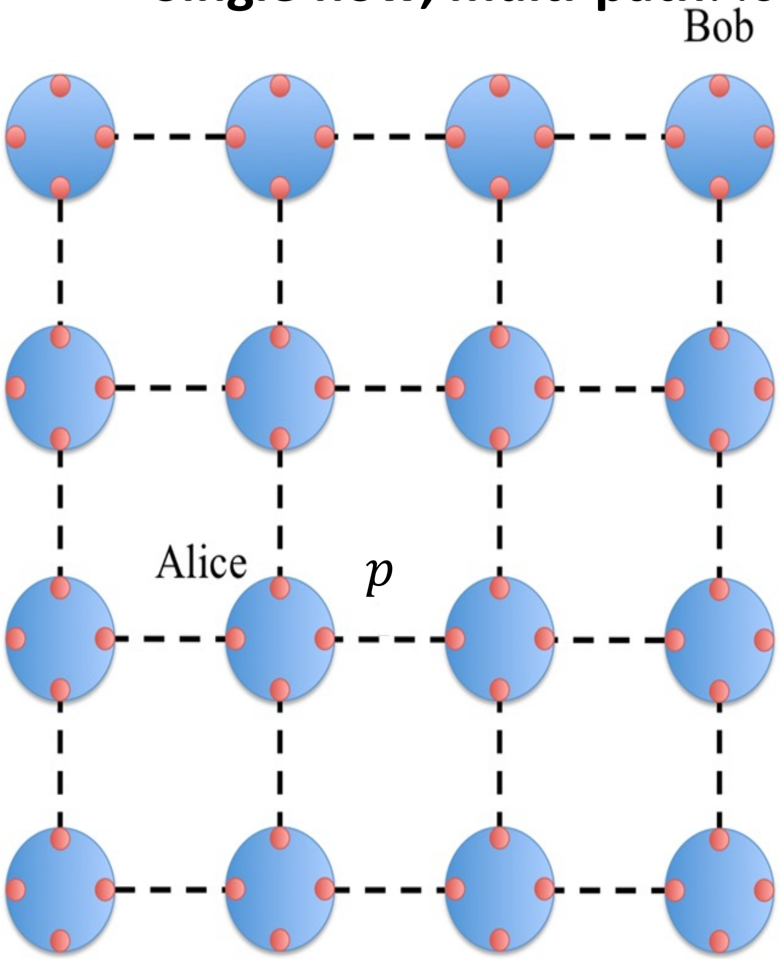
Routing & multipath diversity

Multi-path entanglement routing

- Optimal local connection rules for the repeater nodes?
  - **Single flow, multi-path**: local vs. global link state information



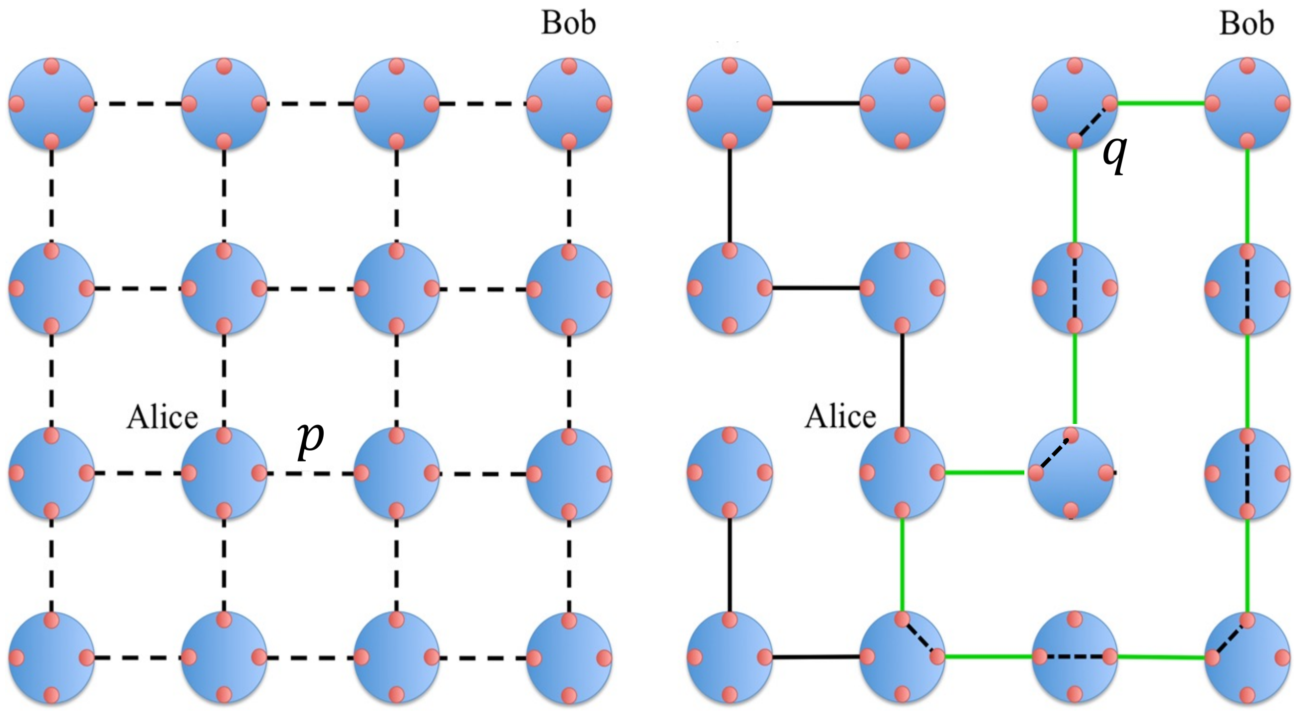
p – link Bell state success probability

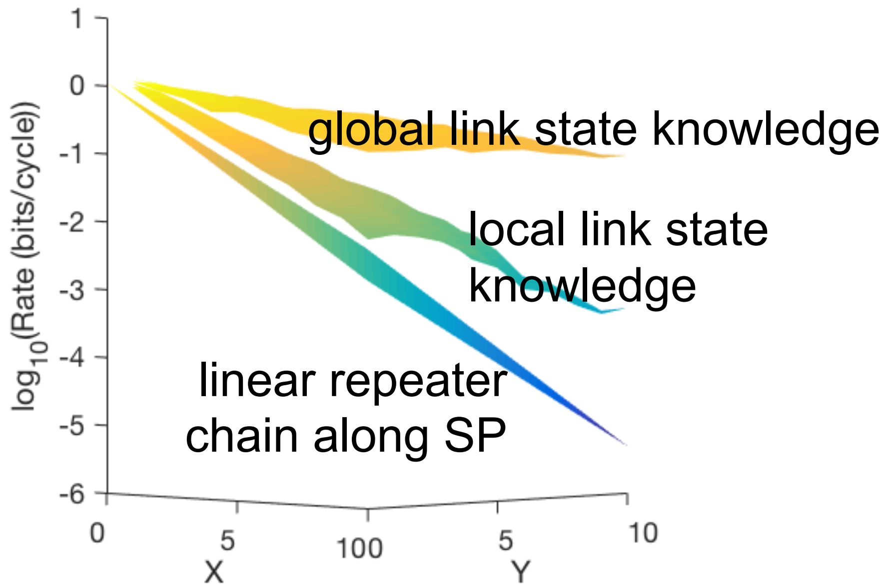
q – Bell swap success probability

M. Pant, etal, Nature NPJ Quantum Information (2019)

Multi-path entanglement routing

- Optimal local connection rules for the repeater nodes?
  - **Single flow, multi-path**: local vs. global link state information

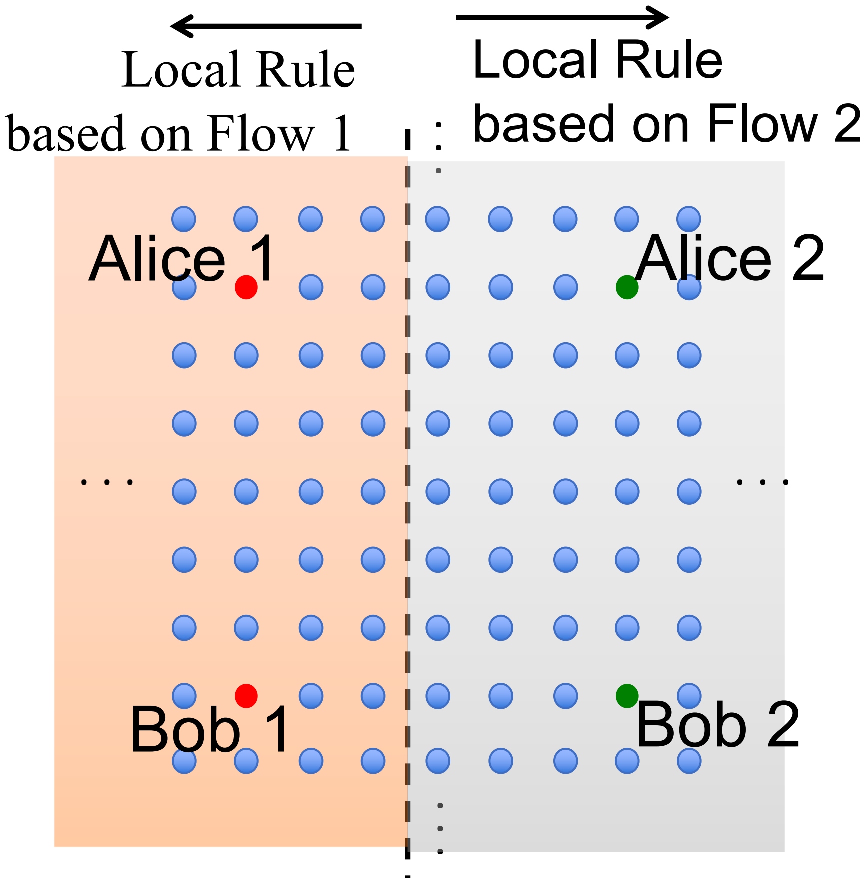


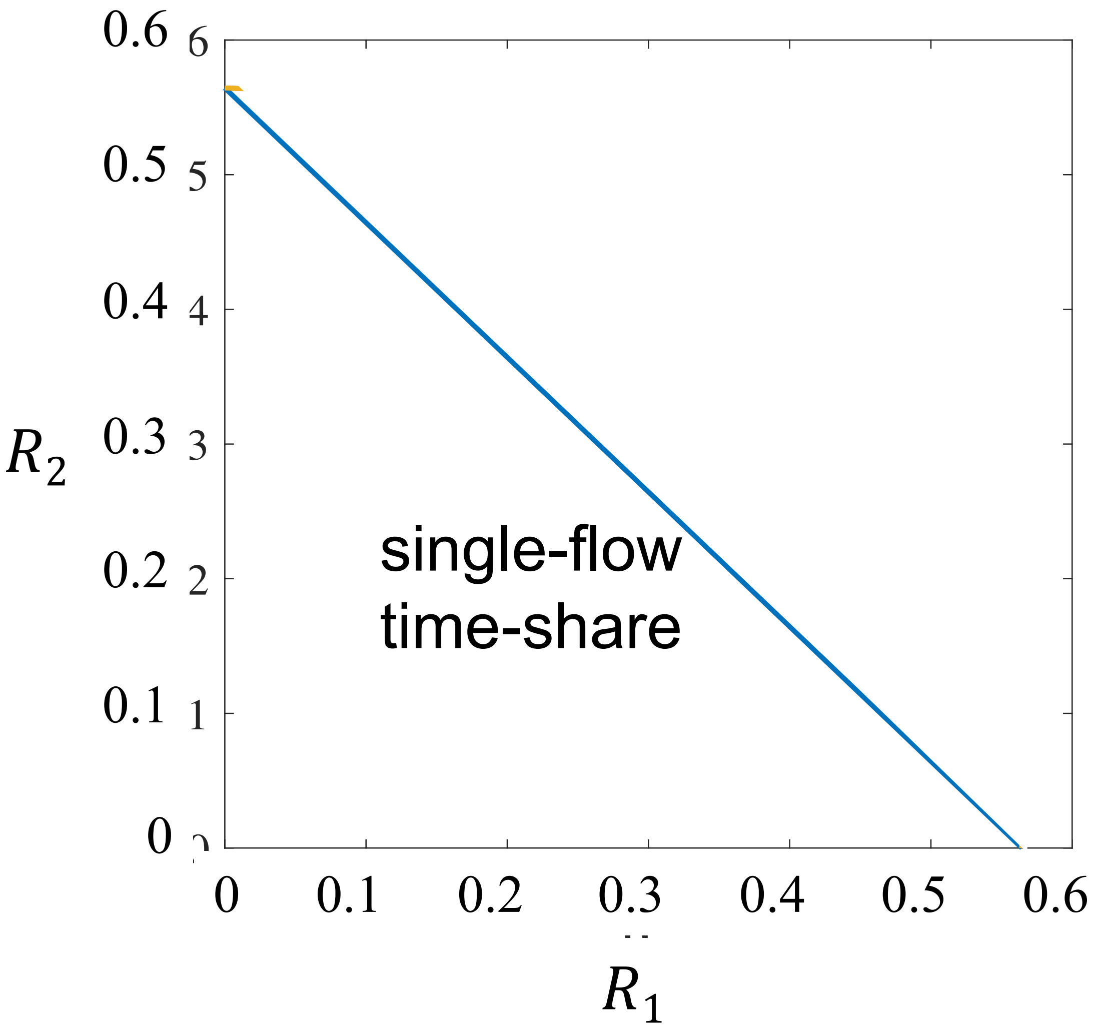


Even with only local information, Multi-path routing over 2D repeater network outperforms linear repeater chain.
Still exponential decay

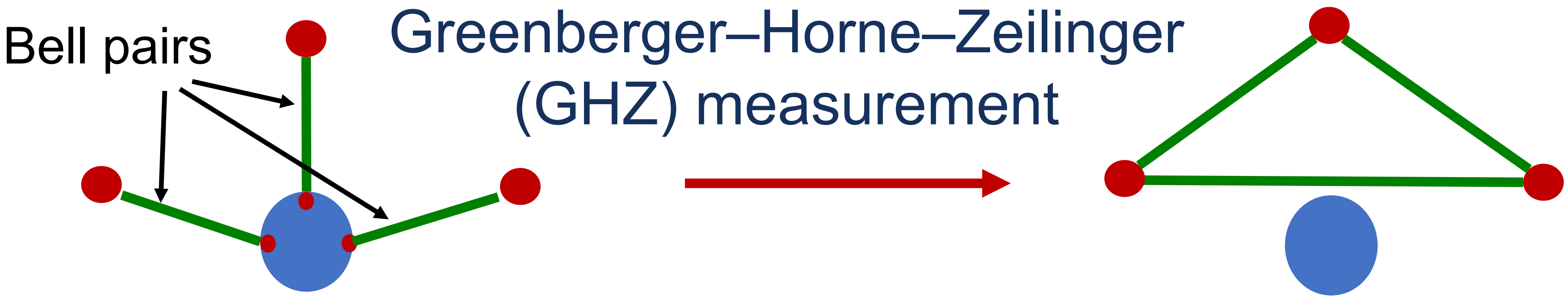
M. Pant, etal, Nature NPJ Quantum Information (2019)

Multi-flow routing





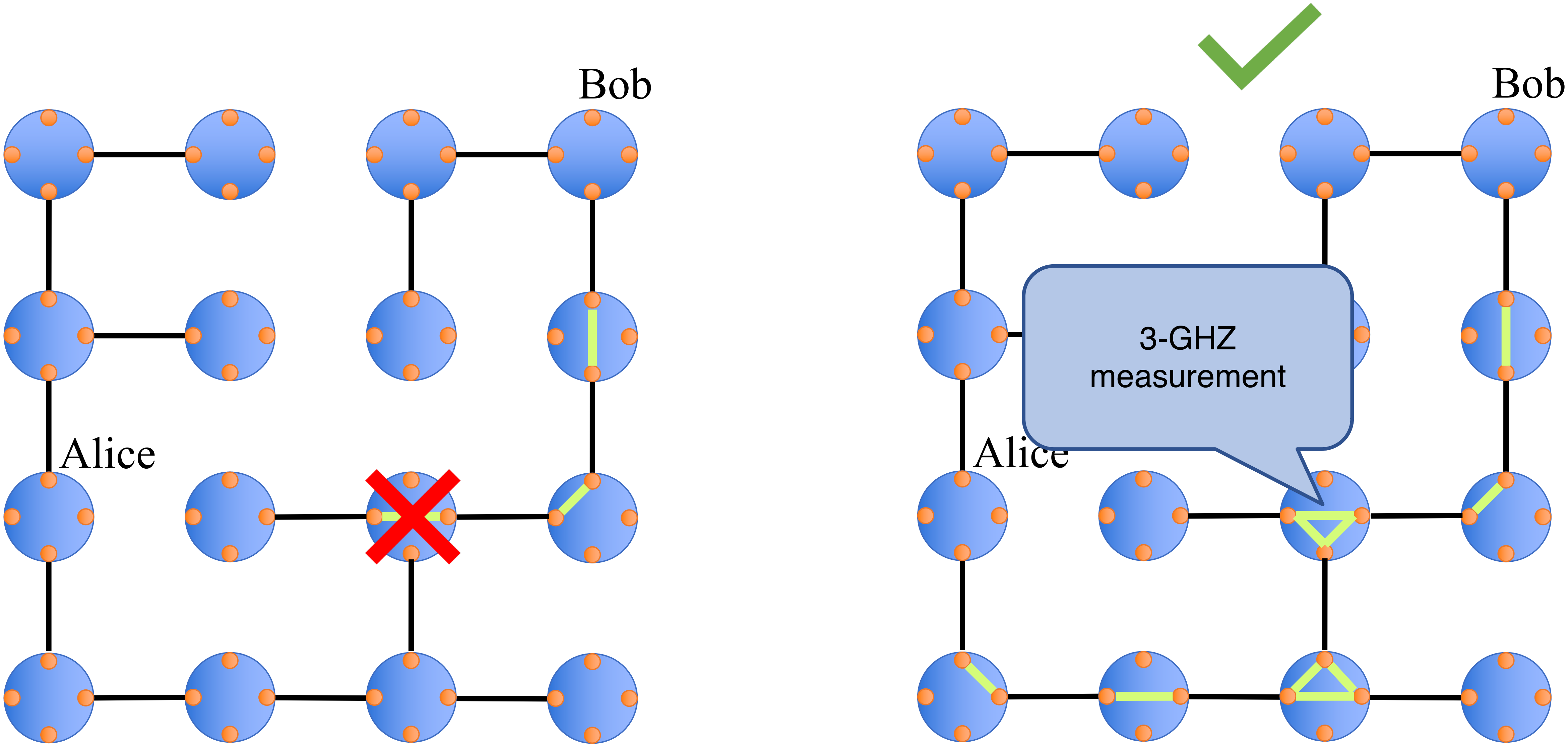
Can we achieve distance independent rates?



n-partite GHZ state $|GHZ\rangle = \frac{|00\cdots0\rangle+|11\cdots1\rangle}{\sqrt{2}}$

- used in multiparty QKD, secret sharing, quantum sensing, …

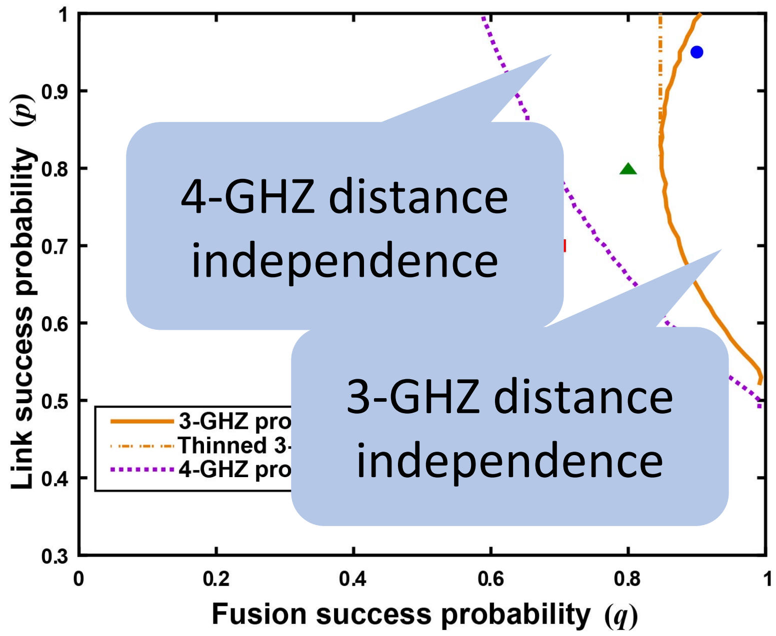
When GHZ measurement helps

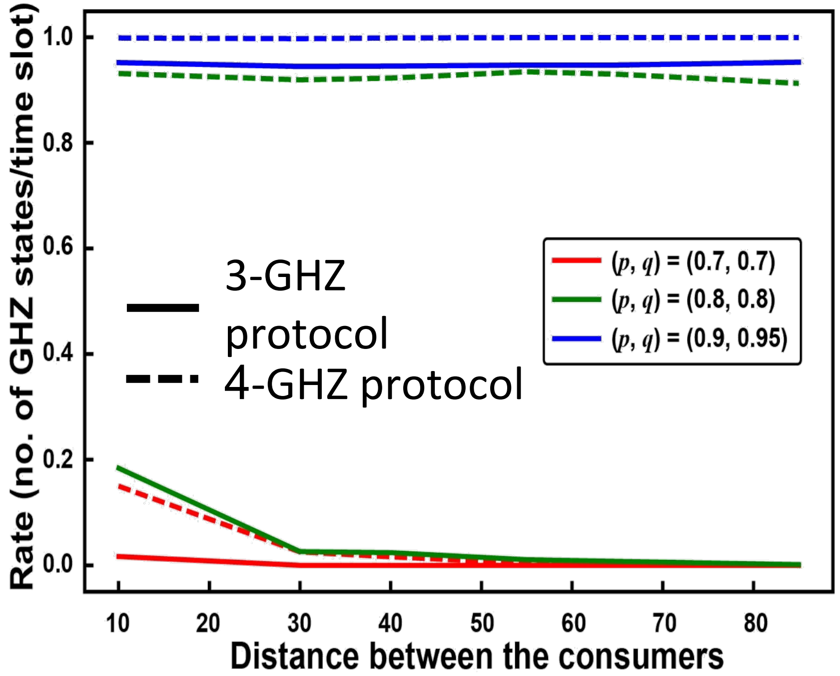


Rate vs. distance

- 3-GHZ protocol
  - measures up to 3 entangled links
  - randomly selects 3 entangled links in presence of 4 entangled links
- 4-GHZ protocol
  - measures up to 4 entangled links
- Maps to a site/bond percolation problem
  - distance independence occurs when system percolates

Both achieve distance independent rates (with one memory)





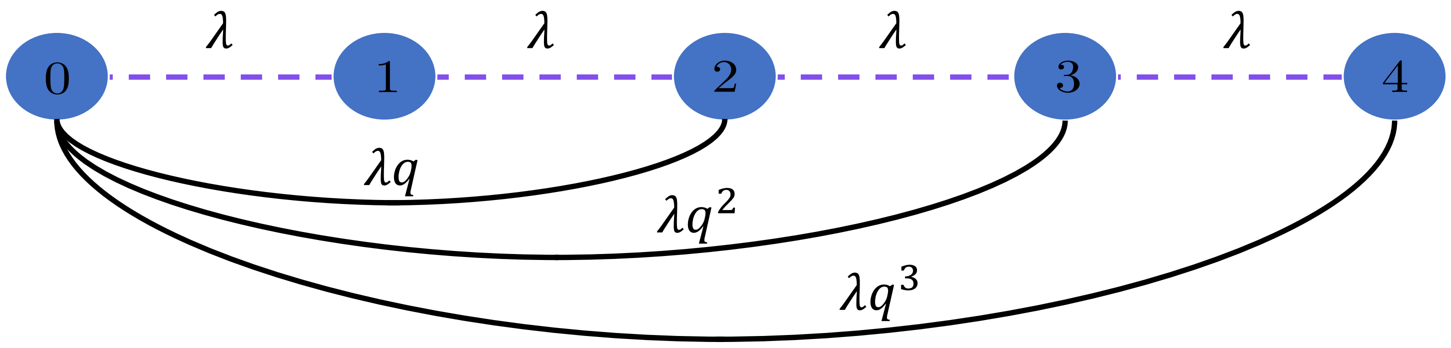
Challenges

- Accounting for noise
- Designing efficient protocol to transmit classical bits to end-nodes
- Nodes have four interfaces – how can these be taken advantage of to increase rate?
- Sharing a network among multiple users

Flow and swap optimization

Scheduling entanglement swaps

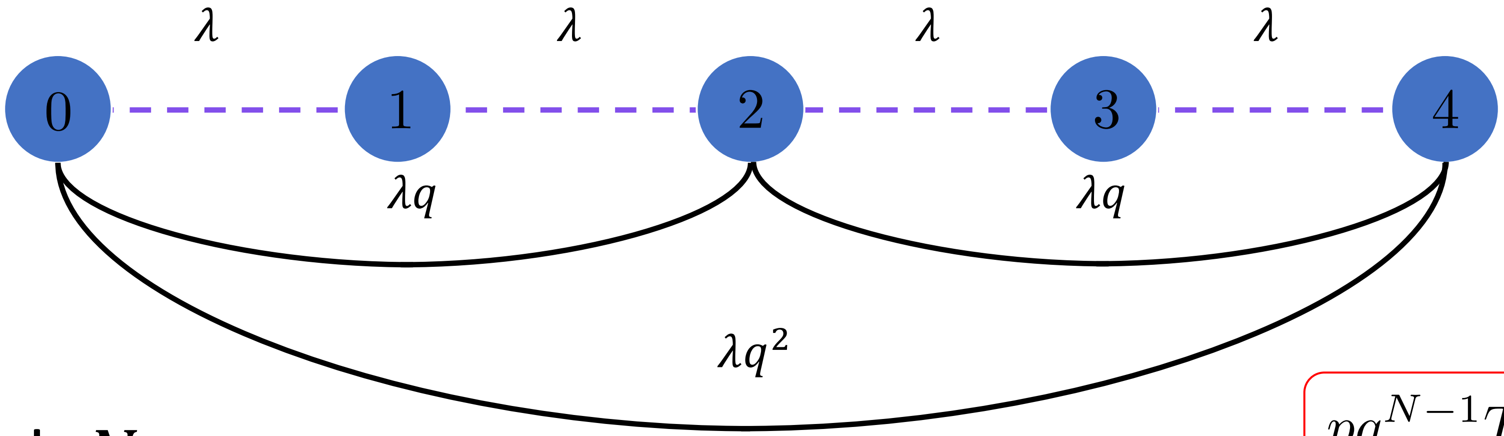
- repeaters not perfect; Bell state measurement success probability: $q < 1$
- sample schedule: link Bell pair generation rate λ



- operations can be executed *in any order*
- capacity decays exponentially in number of repeaters

Entanglement swap scheduling

- repeaters not perfect; Bell state measurement success probability: $q < 1$



Path of length N

$pq^{N-1}T$ vs. $pq^{\log_2 N}T$

- nested entanglement swapping: $\lambda q^{\log N}$

Notice behavior when $q = 1$

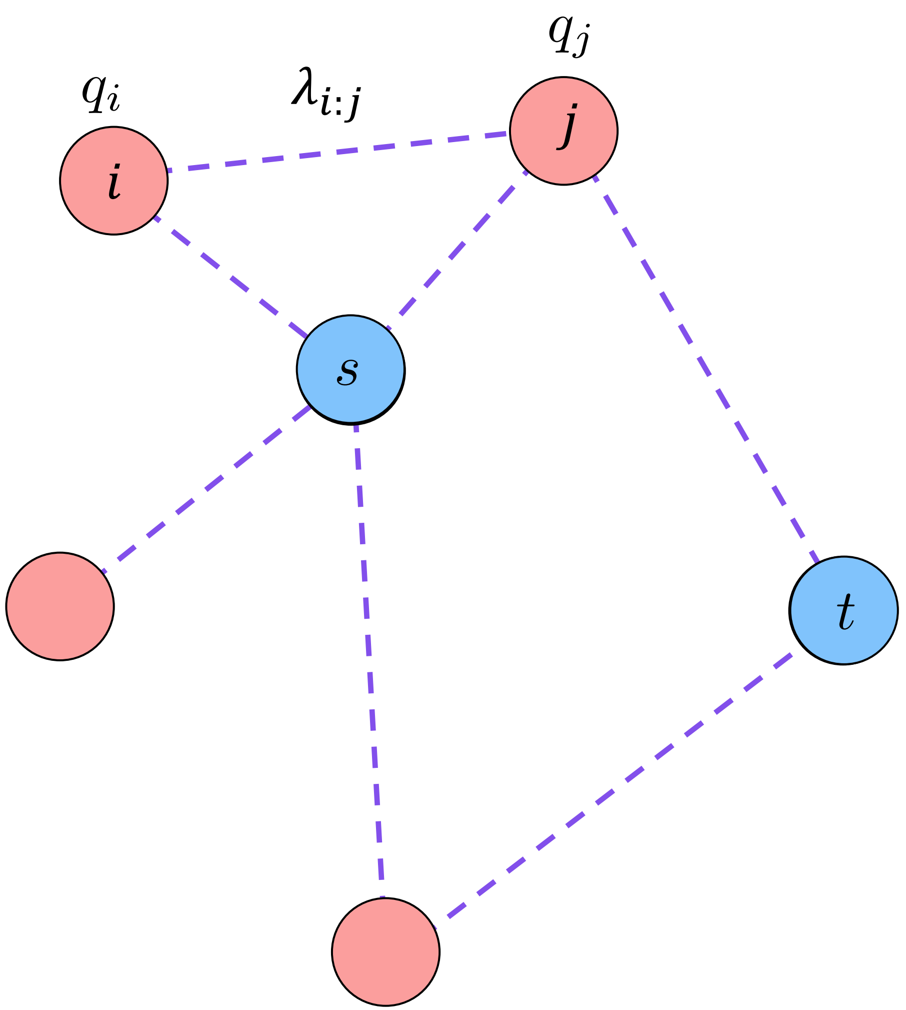
Entanglement scheduling affects performance!

W. Dai, etal., *IEEE TQE*, 2020

Problem Formulation

More generally, consider a network consisting of switches and channels $(\mathcal{N}, \mathcal{E})$

- known link Bell pair generation rates
  $\lambda_{i:j}, (i, j) \in \mathcal{E}$
- known success swap probabilities
  $q_i, i \in \mathcal{N}$
- two switches chosen as end nodes desiring entanglement



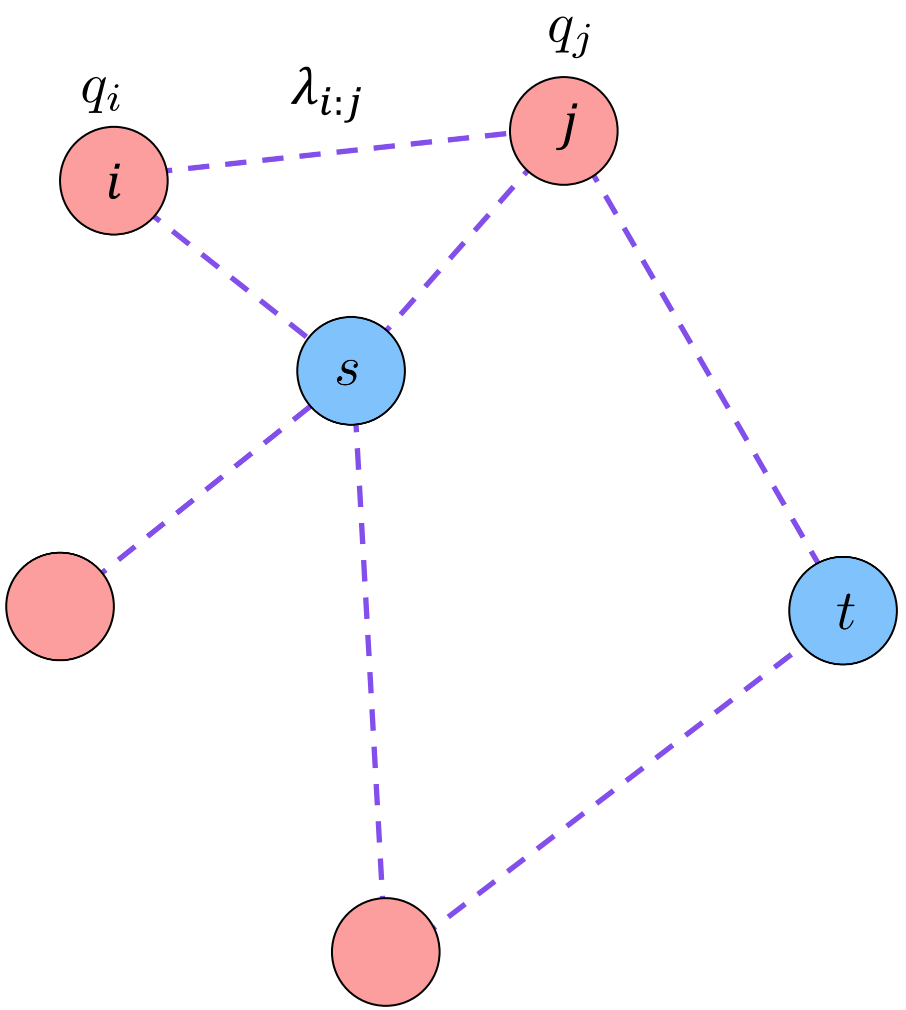
Problem Formulation

Time is slotted; each slot divided into two phases:

- Phase I: entanglement generation
- Phase II: entanglement swapping

Performance metric: entanglement distribution rate

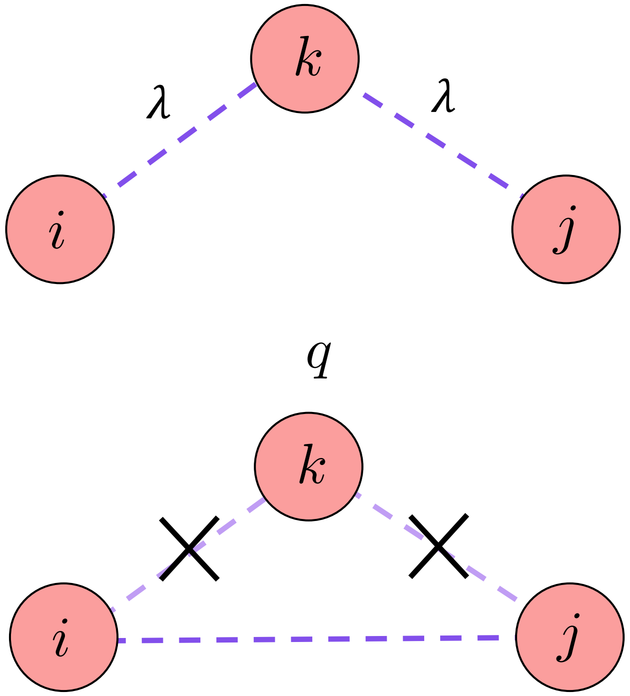
$$\lambda = \lim_{T\to\infty} \frac{\text{number of } |\Psi_{st}\rangle \text{in the first } T \text{ slots}}{T}$$

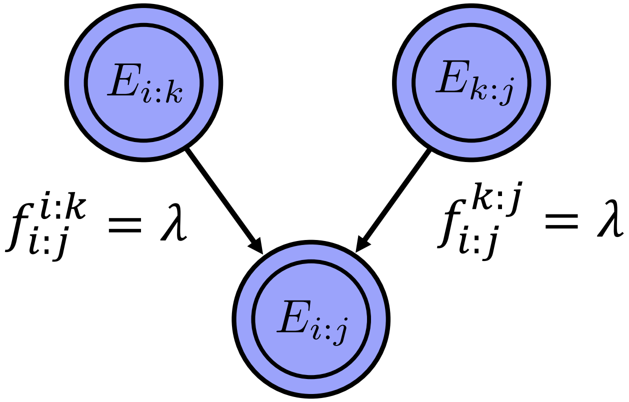


E-nodes and E-flows

Idea: quantum network + protocol → new graph

- E-nodes represent qubit pairs
- E-flows represent rate of entanglement exchanged among E-nodes (determined by channels and protocols)





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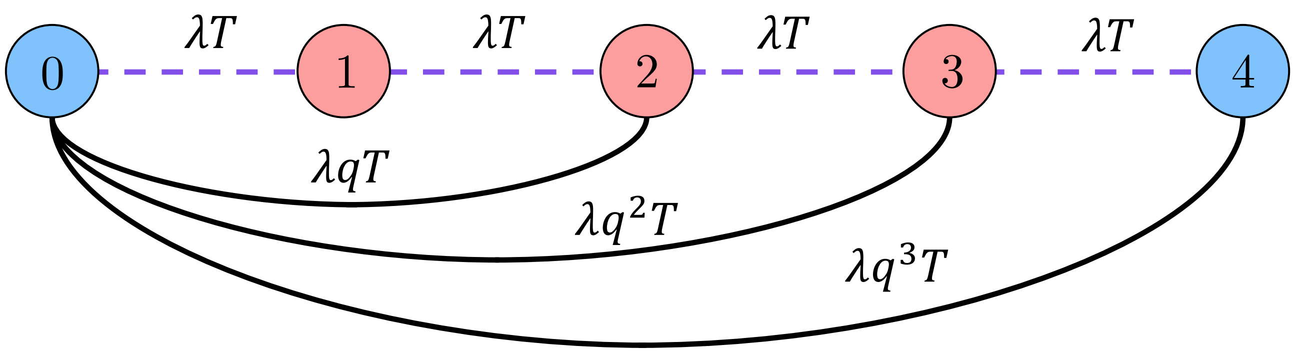
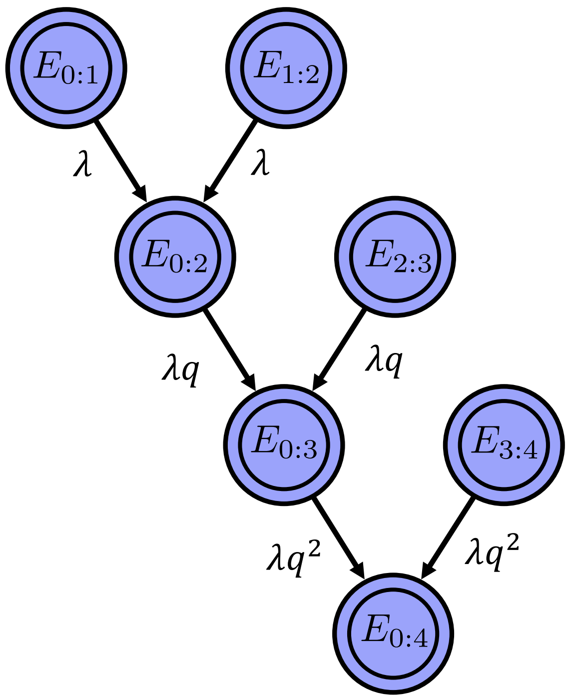
E-nodes and E-flows

Idea: quantum network + protocol → new graph

- E-nodes represent the qubit pairs
- E-flows represent the rate of entanglement exchange among E-nodes (determined by entanglement swapping protocol)





Optimization Problem

Theorem: (DaiPengWin) For a given network $\{\lambda_{a:b}\}_{(a,b)\in\mathcal{E}}, \{q_c\}_{c\in\mathcal{N}}$, the optimal entanglement distribution rate is the solution to

$$
\begin{aligned}
\underset{\substack{\{f_{i:j}^{i:k}: i,j,k\in\mathcal{N}\} \\ \{u_{i:j}\}_{(i,j)\in\mathcal{E}}}}{\text{maximize}} \quad & \lambda_{s:t}\ 1_{\mathcal{E}}(s,t) + \sum_{k\in\mathcal{N}\setminus\{s,t\}} q_k \frac{f_{s:t}^{s:k} + f_{s:t}^{k:t}}{2} \\
\text{subject to} \quad & u_{i:j}\ \lambda_{i:j} 1_{\mathcal{E}}(i,j) + \sum_{k\in\mathcal{N}\setminus\{i,j\}} q_k \frac{f_{i:j}^{i:k} + f_{i:j}^{k:j}}{2} = \sum_{k\in\mathcal{N}\setminus\{i,j\}} (f_{i:k}^{i:j} + f_{k:j}^{i:j}), \quad i,j\in\mathcal{N}, \{i,j\}\neq\{s,t\} \\
& f_{i:j}^{i:k} = f_{i:j}^{k:j} \geq 0, \qquad i,j,k\in\mathcal{N} \\
& f_{s:k}^{s:t} = f_{k:t}^{s:t} = 0, \qquad k\in\mathcal{N} \\
& 0 \leq u_{i:j} \leq 1, \qquad (i,j)\in\mathcal{E}.
\end{aligned}
$$

Optimization Problem

Theorem: (DaiPengWin) for a given network $\{\lambda_{a:b}\}_{(a,b)\in\mathcal{E}}, \{q_c\}_{c\in\mathcal{N}}$, the optimal entanglement distribution rate is the solution to:

maximize
E-flow related quantities
subject to

entanglement distributed between source and sink nodes

dynamic equilibrium for each E-node

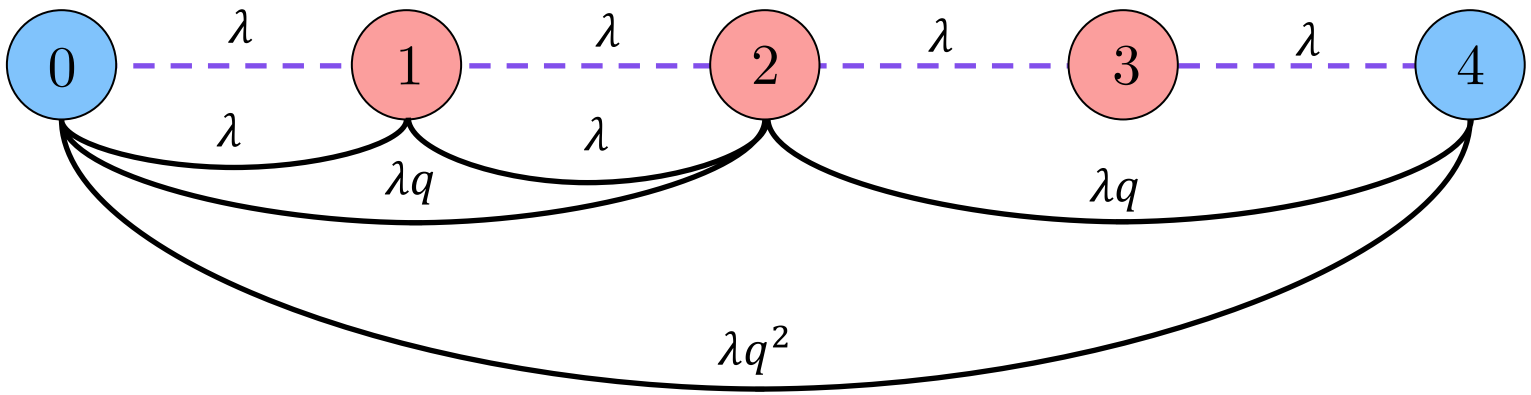
constraints on each E-flow quantity

Remark:

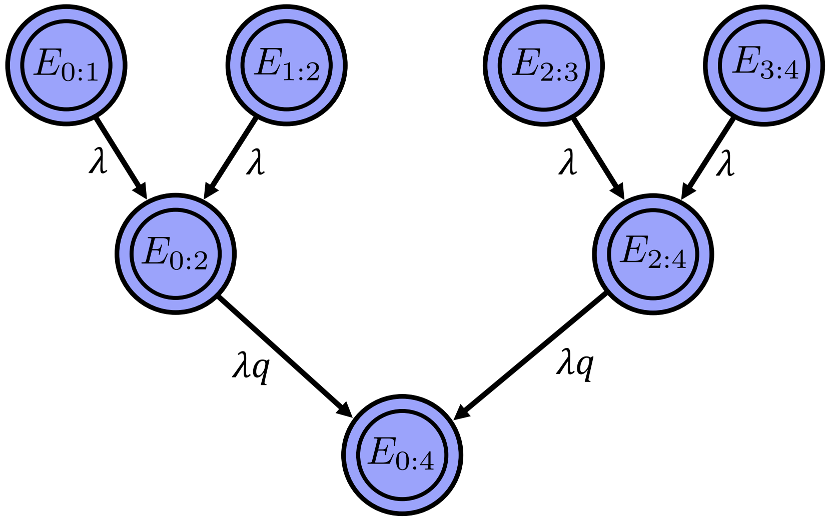
- linear programming problem with complexity $poly(|\mathcal{N}|)$
- protocol that achieves the optimal rate

Example of An Optimal Solution

Homogeneous repeater chains



E-nodes and E-flows



Closed-form Solution

Theorem: (DaiPengWin) For homogeneous repeater chains with an even number, N, of segments, maximal entanglement distribution rate is

$$R(N) = \frac{N\lambda q^{n+1}}{N(1-q)+2^{n}(2q-1)}$$

where $n = \lceil \log_2 N \rceil - 1$. Similar result for N odd

Remarks

- polynomial decay with respect to N and distance L

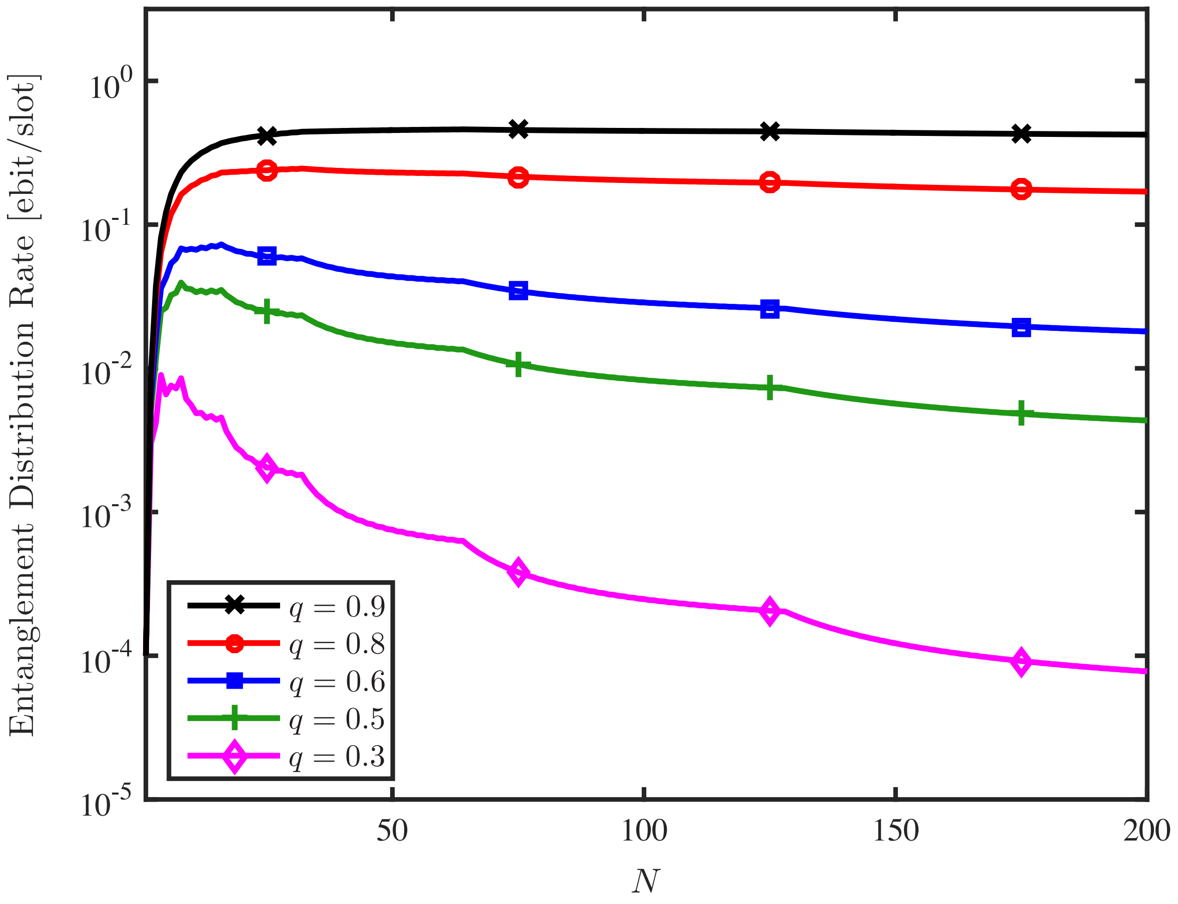
$$R(N) \sim O(L^{\log q})$$

- contrast to subexponential decay $O(e^{-t\sqrt{\alpha L}})$

Subexponential rate versus distance with time-multiplexed quantum repeaters, PRA 104, 052612, 2021

Homogeneous Repeater Chain

- Total distance $L = D \cdot N = 200$ km (fixed)
- Request rate $\lambda_{i:j} = 10^{-\gamma D/10}$; $\gamma = 0.2$ dB/km



Challenges

- Extension to multiple users
- Handling noise
  - maximizing entanglement subject to minimum fidelity constraint
  - introducing purification as part of optimization
- Introducing memory constraints

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Network Management: Quantum Network Tomography

Outline

NETWORK MANAGEMENT AND TOMOGRAPHY OVERVIEW

CLASSICAL NETWORK TOMOGRAPHY

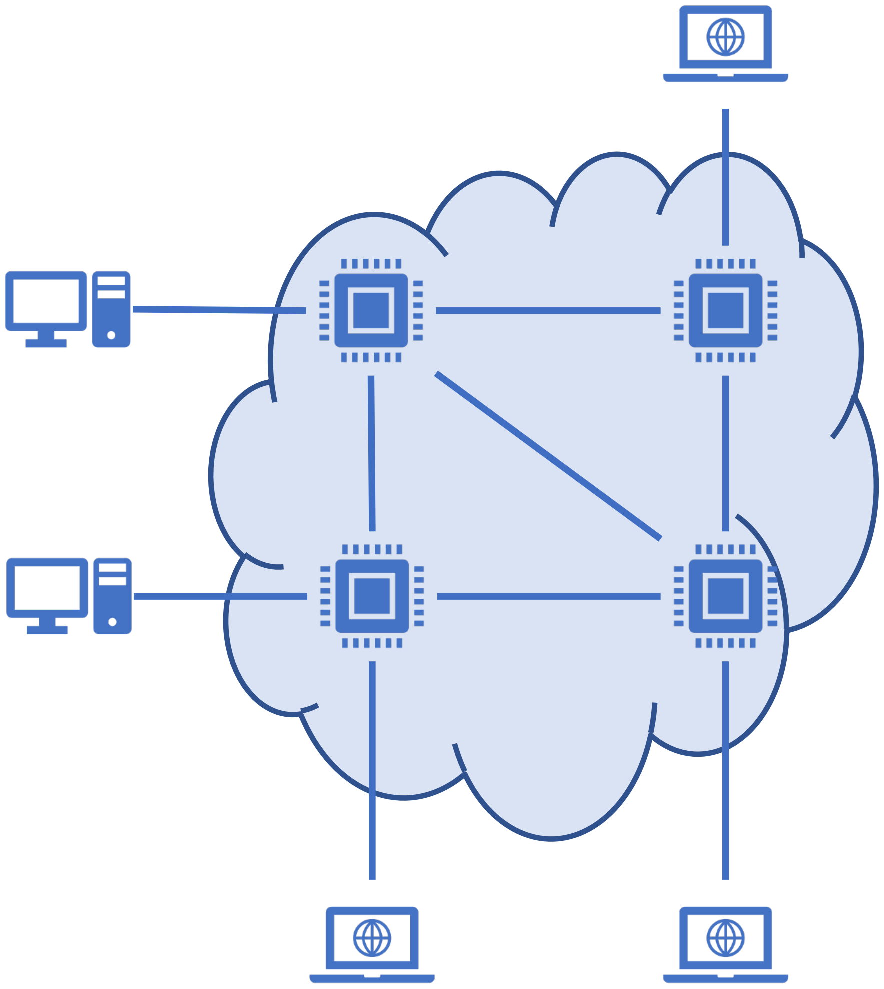
QUANTUM NETWORK TOMOGRAPHY (QNT)

STATE DISTRIBUTION FOR QNT

CHARACTERIZING STAR NETWORKS

Network management

- Network component data collection
- Information to aid decision making
- Fault-detection for hardware / software
- Determine traffic patterns



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Network tomography

Goal

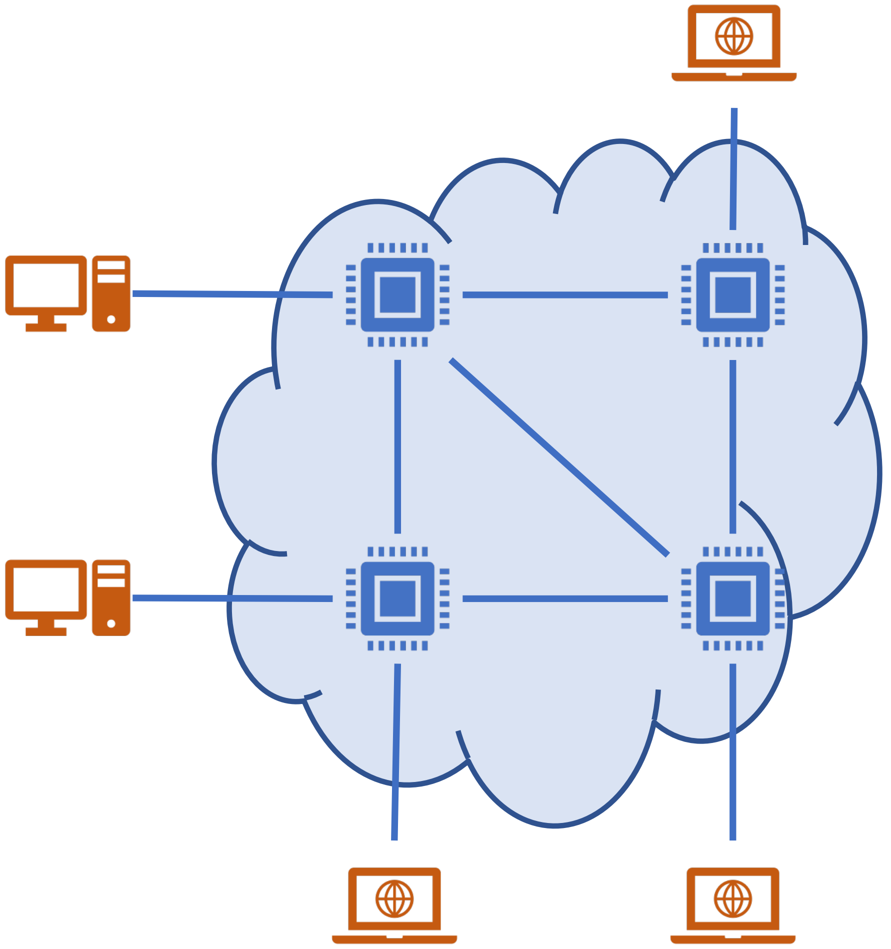
Infer internal behavior in network from external nodes

In practice

Estimate error parameters for internal components from end-to-end measures

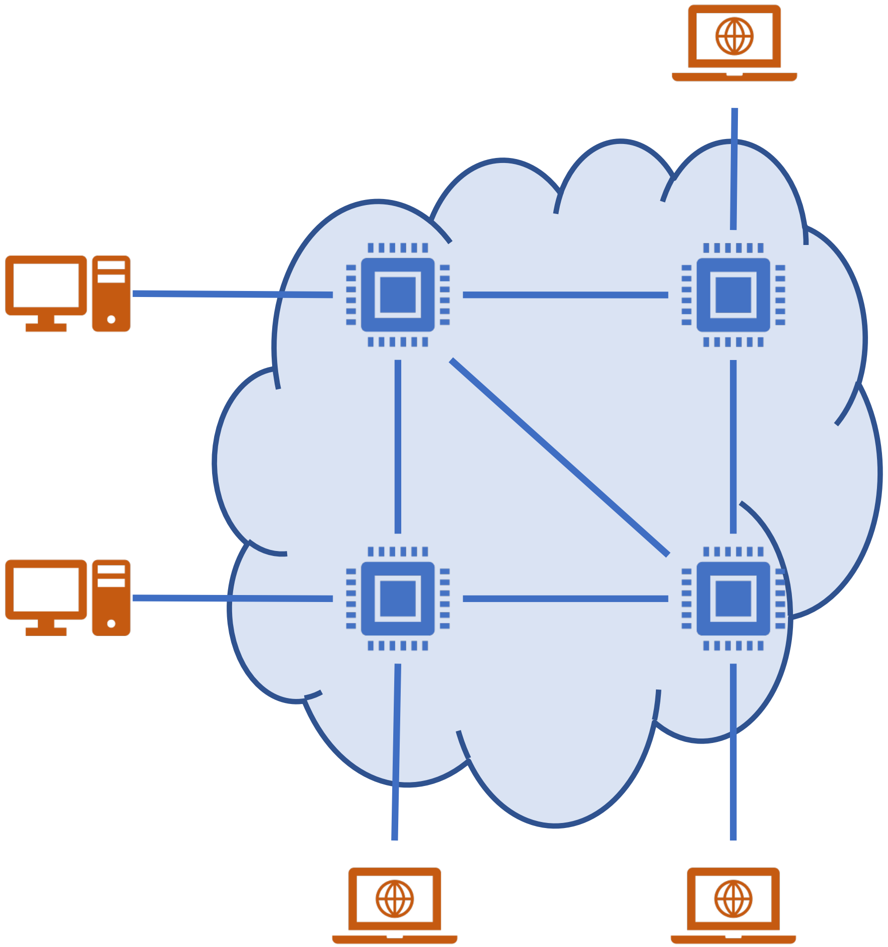
Identifiability

Obtain one value for parameters given a set of observations



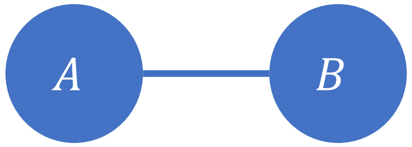
Why end-to-end?

- No participation by network needed
  - Measurement probes regular packets
- No administrative access needed
- Inference across multiple domains
  - No cooperation required
  - Monitor service level agreements
- Reconfigurable applications
  - Video, audio, reliable multicast



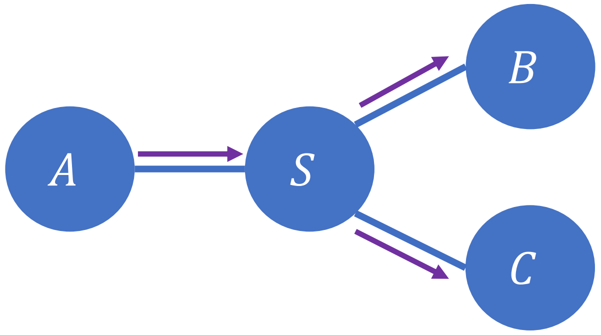
Definitions

Link-level metrics



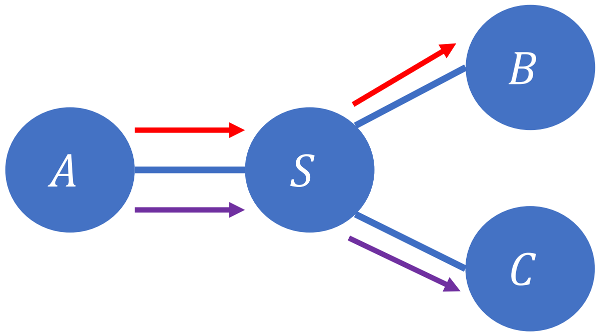
E.g: delay, loss, bit-flip rate

Multicast communication



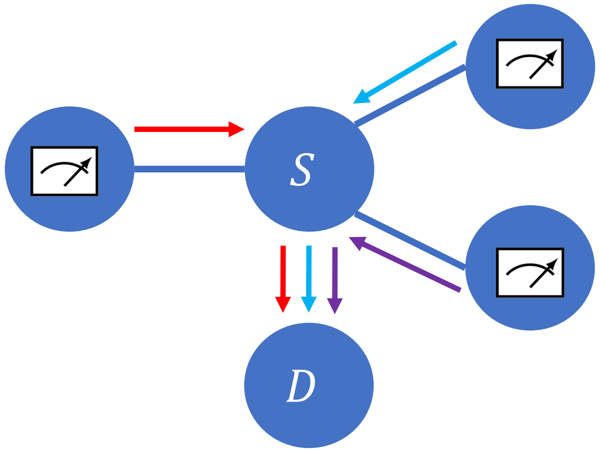
One-to-many

Unicast communication



One-to-one

Estimation



Data sent to fusion center

Bidirectional Unicast Tomography

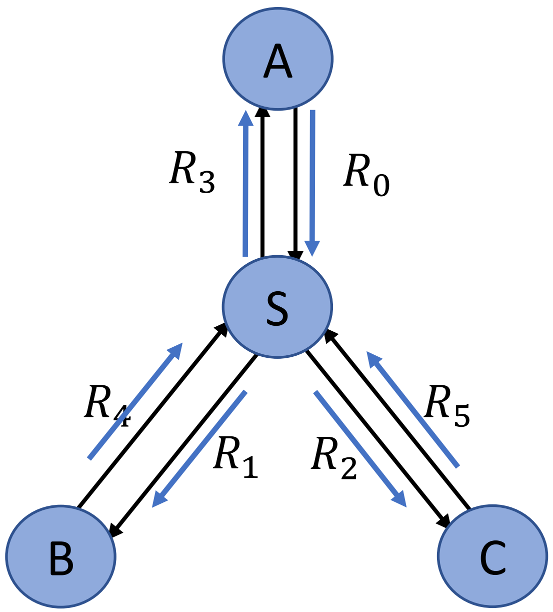
Assumptions

- Links are asymmetric
- Additive metrics

Results

- 6 equations, 6 unknowns
- Not linearly independent
  - Not identifiable

$$R_{AB} = R_0 + R_1$$
$$R_{BA} = R_4 + R_3$$
$$R_{AC} = R_0 + R_2$$
$$R_{CA} = R_5 + R_3$$
$$R_{BC} = R_4 + R_2$$
$$R_{CB} = R_5 + R_1$$



Round-trip Unicast Tomography

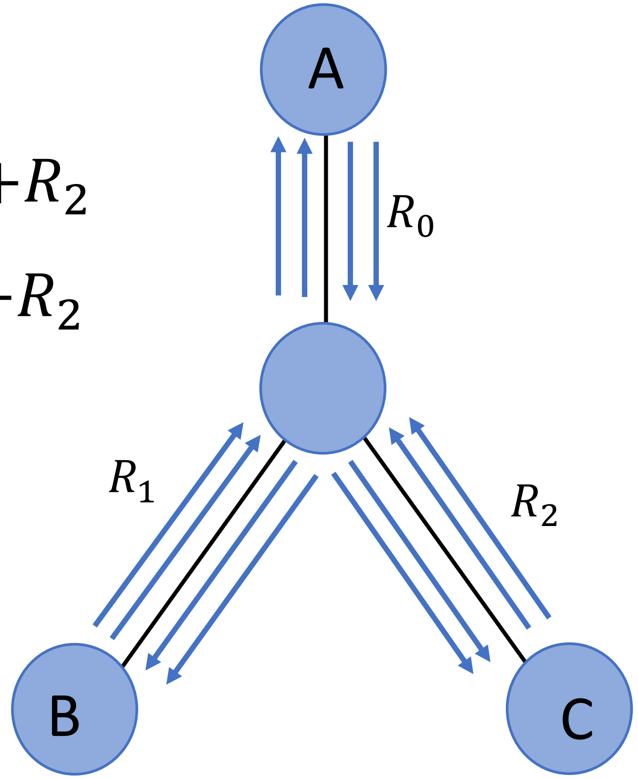
Assumptions

- Links are symmetric
- Additive metrics

Results

- Linear independence! (identifiable)
- True for general trees
- Can infer some link delays within general graph
- Measurements over cycles

$$R_{AB} = R_0 + R_1$$
$$R_{AC} = R_0 + \qquad + R_2$$
$$R_{BC} = \qquad R_1 + R_2$$



Bottom Line

- Similar approach for losses
- Yields round trip and one way metrics for subset of links
- Approximations for other links
  - choose delays to
    - minimize MSE
    - maximize entropy

Unicast Tomography Poll

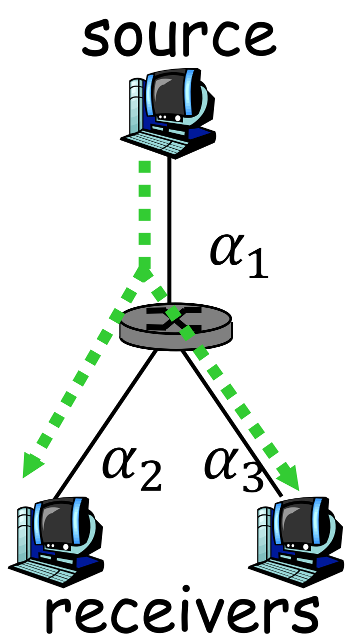
- What is a sufficient condition for link identifiability through unicast tomography?
  - Link asymmetry
  - Link symmetry
  - Invertibility of routing matrix
  - Star network topology

Answer

- What is sufficient for link identifiability through unicast tomography?
  - Link asymmetry
  - Link symmetry
  - **Invertibility of routing matrix**
  - Star network topology

MINC (Multicast Inference of Network Characteristics)

- multicast probes
  - copies made *as needed* within network
- receivers observe correlated performance
- *exploit* correlation to get link behavior
  - loss rates
  - delays



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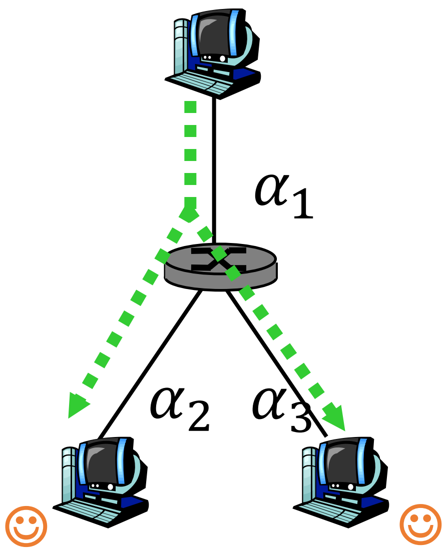

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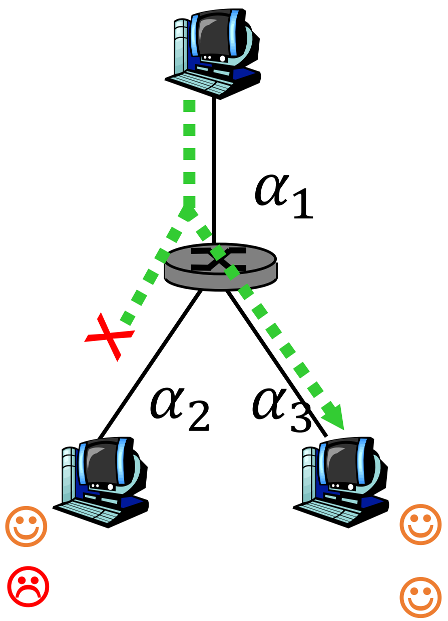

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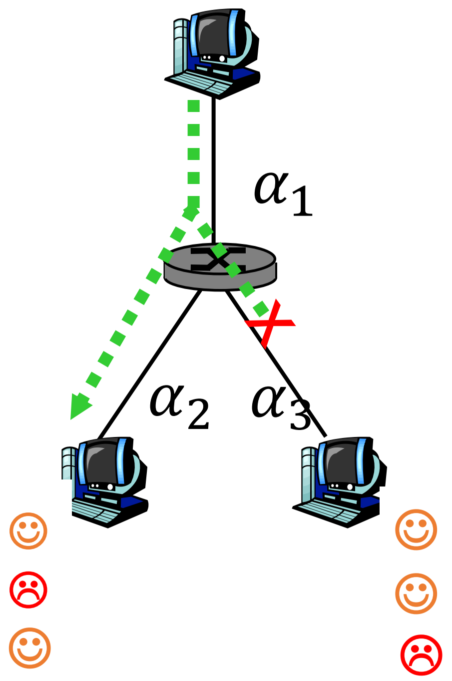

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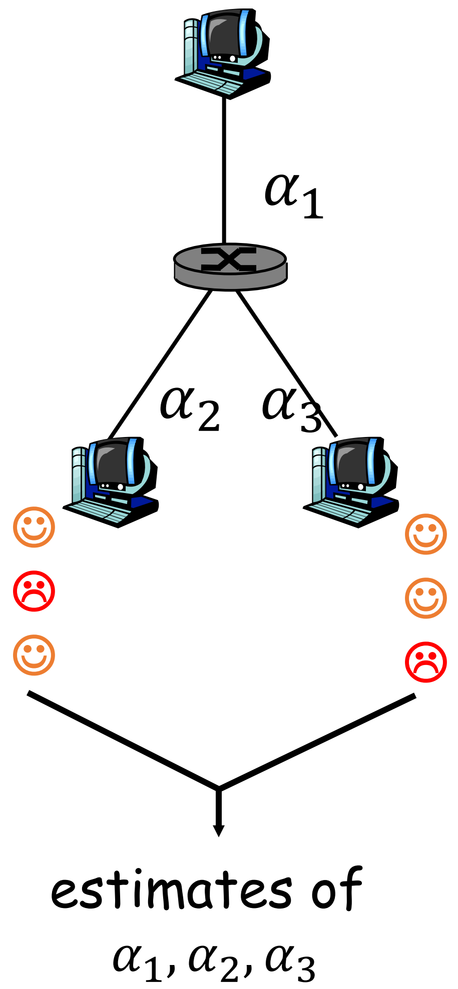
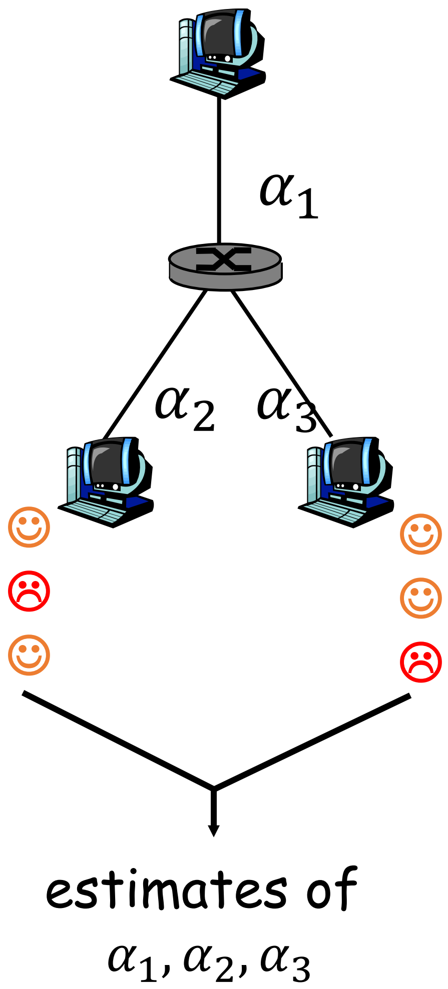
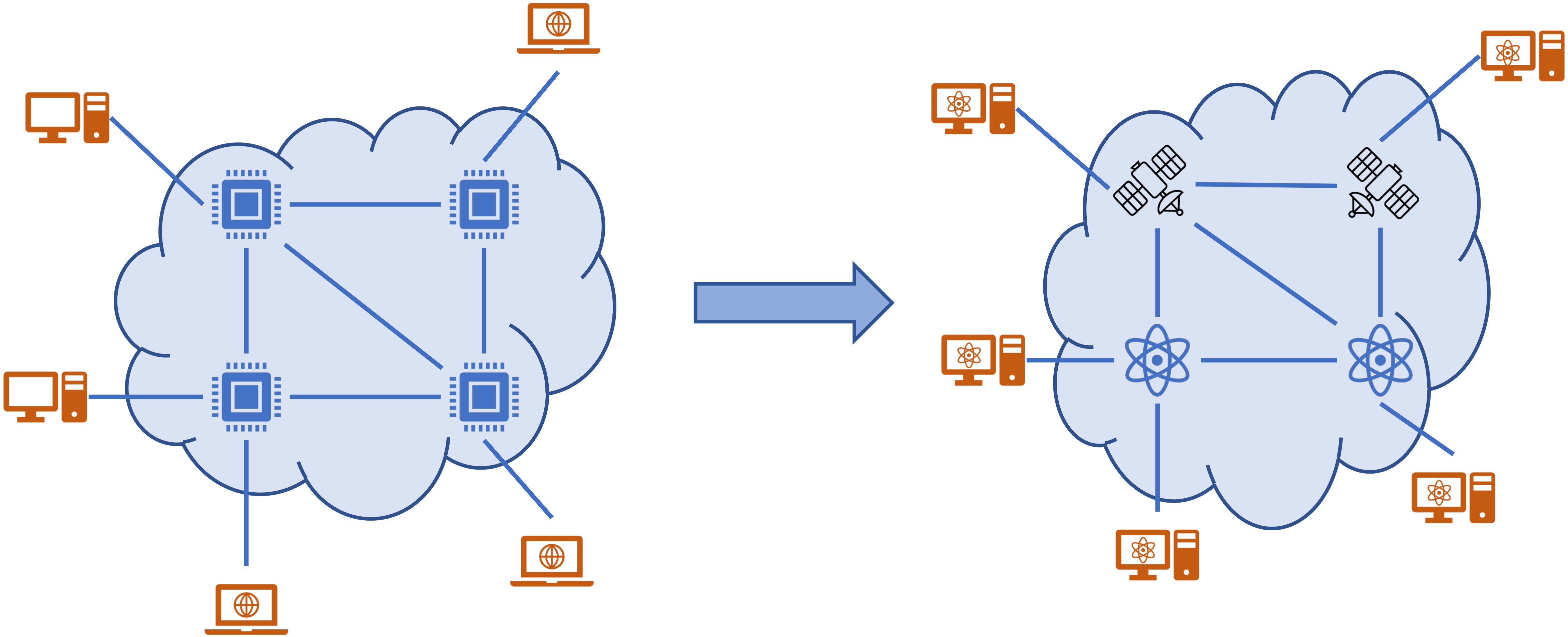


Bottom Line

- Binary tree identifiable
- Correlation allows identification of loss in links
- Different network utilization than unicast

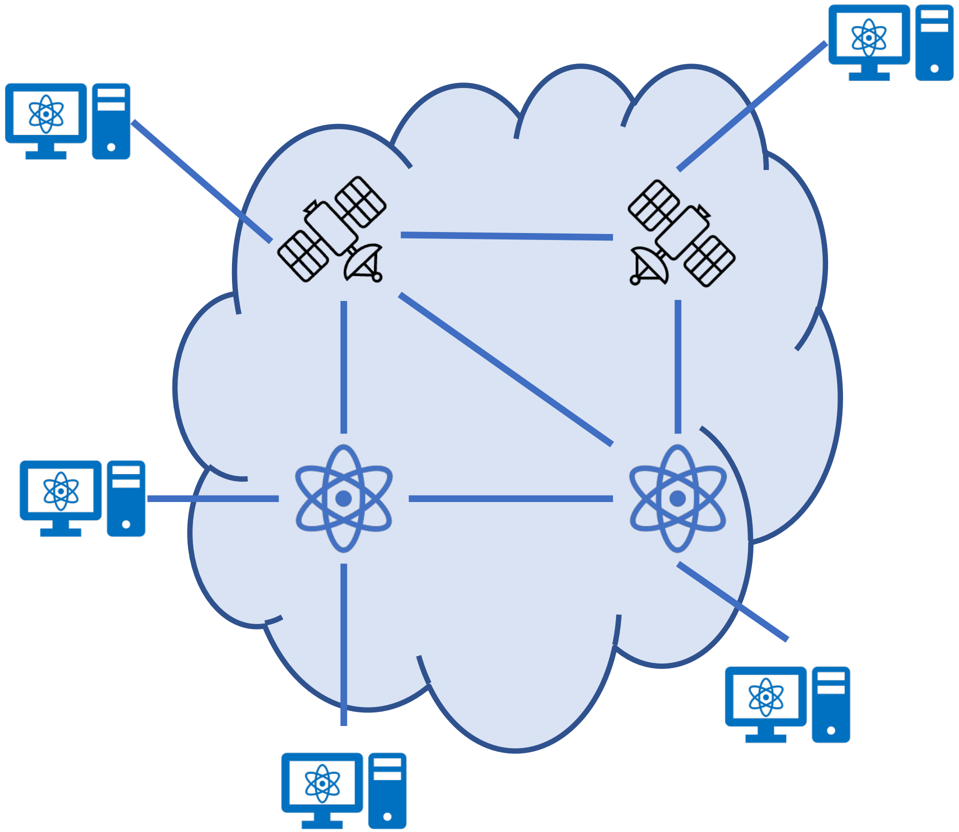


Quantum Network Tomography



Motivation

- Inhomogeneous quantum hardware
- Hybrid communication media
- Network management
  - Faulty network hardware identification
  - Improved decision-making in resource utilization
  - Noise-informed quantum error correction
- Quality assurance
- Reconfigurable applications



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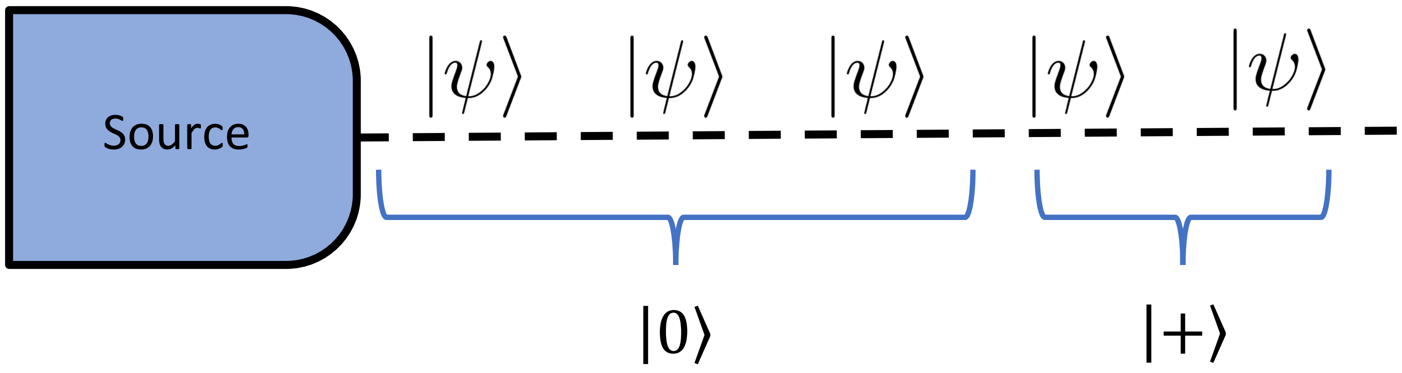
From Classical to Quantum

Classical	Quantum
Link-level metrics	Quantum channel parameters
Probes	State Distribution
Unicast	Bipartite state distribution
Multicast	Multipartite state distribution
End-to-end measurements	Measurements in end-nodes

Background: Mixed states

- Pure states
  - Describe closed quantum systems
  - Efficiently represented by unit-norm vectors in complex (Hilbert) space
- Mixed states: statistical ensemble of quantum states
  - E.g Qubit preparation device $60\%|0\rangle, 40\%|+\rangle$
  - Efficiently represented by density matrices



Background: Density Matrices

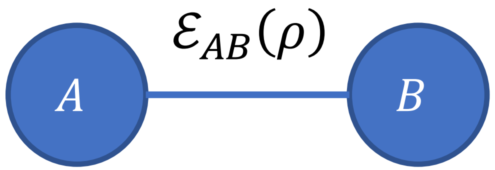
- Suppose one qubit
- If pure state: $|\psi\rangle \in \mathcal{H}^2$, $|\psi\rangle\langle\psi| \in \mathcal{H}^2 \to \mathcal{H}^2$ projector
- If mixed state: $\rho \in \mathcal{H}^2 \to \mathcal{H}^2$
  - $\rho = \sum p_k |\psi_k\rangle\langle\psi_k|$ where $\mathrm{p_k}$ probabilities and $|\psi_k\rangle$ pure states
  - Hermitian, Positive semi-definite and unit trace

E.g Qubit preparation device $60\%|0\rangle, 40\%|+\rangle$

$$|0\rangle\langle 0| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad |+\rangle\langle +| = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \qquad \rho = \begin{pmatrix} 0.8 & 0.2 \\ 0.2 & 0.2 \end{pmatrix}$$

Single Qubit Pauli Channels

Links represent quantum channels



For all links $e \in E$

$$\mathcal{E}_e(\rho) = \sum_k \theta_{ek} \sigma_k \rho \sigma_k$$

$$\rho, \sigma_k : \mathcal{H}^2 \to \mathcal{H}^2$$

$$\sigma_k \in \{I, X, Y, Z\}$$

$$\theta_{ek} \in \mathbb{R}, \sum_k \theta_{ek} = 1$$

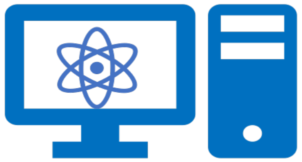
Examples

Bit-flip	$\|0\rangle \to \|1\rangle, \|1\rangle \to \|0\rangle$ $\mathcal{E}_e(\rho) = \theta_e \rho + (1 - \theta_e) X \rho X$
Phase-flip	$\|+\rangle \to \|-\rangle, \|-\rangle \to \|+\rangle$ $\mathcal{E}_e(\rho) = \theta_e \rho + (1 - \theta_e) Z \rho Z$
Bit and phase-flip	$\mathcal{E}_e(\rho) = \theta_{e0} \rho + \theta_{e1} X \rho X + \theta_{e2} Z \rho Z$

Operational Assumptions

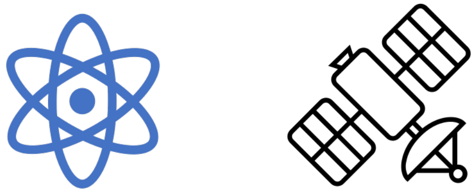
End-nodes V_E

- Perform quantum circuits
- Request network state distribution
- Specify circuits for intermediate nodes



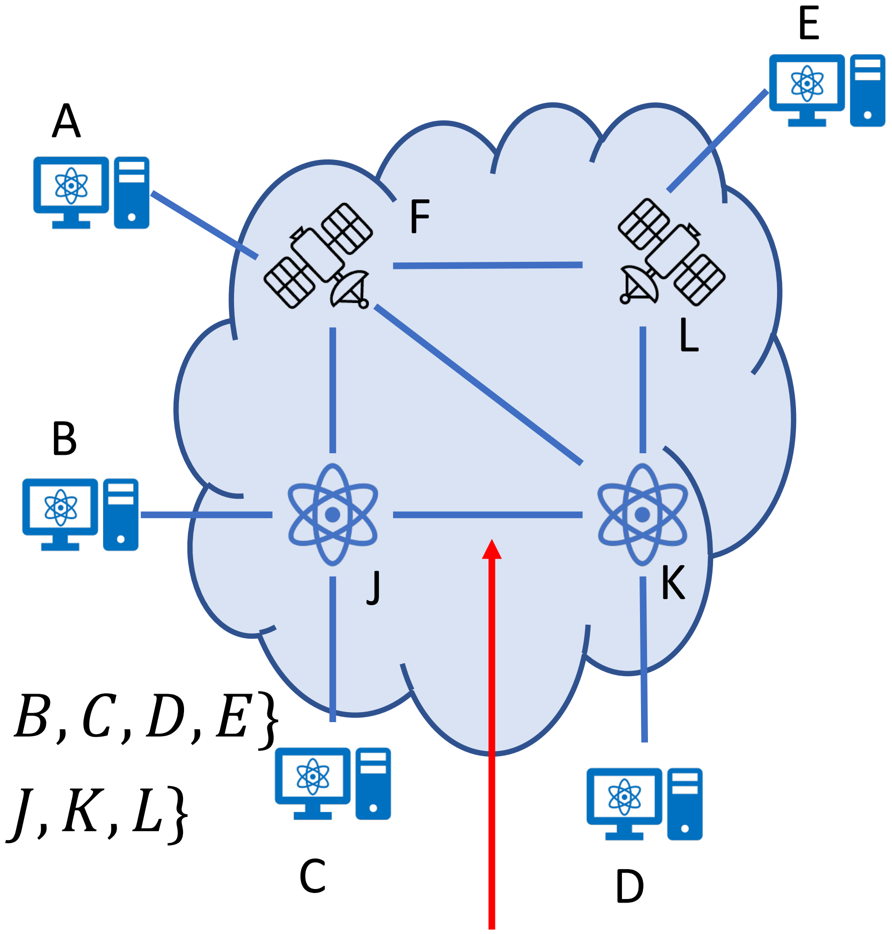
Intermediate nodes V_I

- Receive requests for circuits
- Ancilla qubits
- No measurements for estimation



Quantum Network Model

- Network is graph $G = (V, E)$
  - V : quantum processors
  - E : fiber optics, free space channels
- End and intermediate nodes
- Links: single-qubit quantum channels
- Parametric description for channels
- One-way quantum transmission



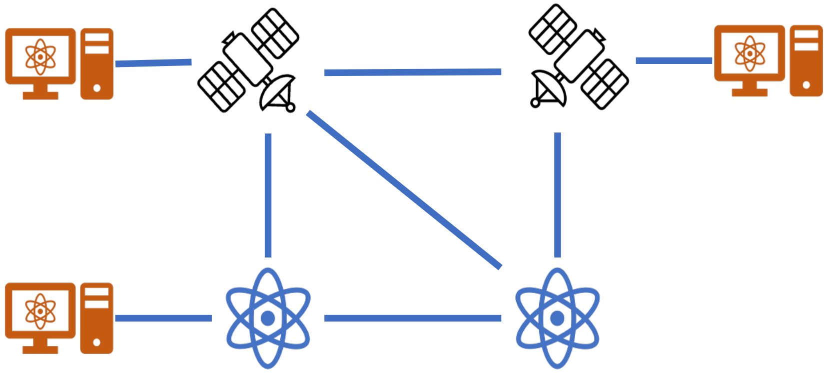
$$V_E = \{A, B, C, D, E\}$$

$$V_I = \{F, J, K, L\}$$

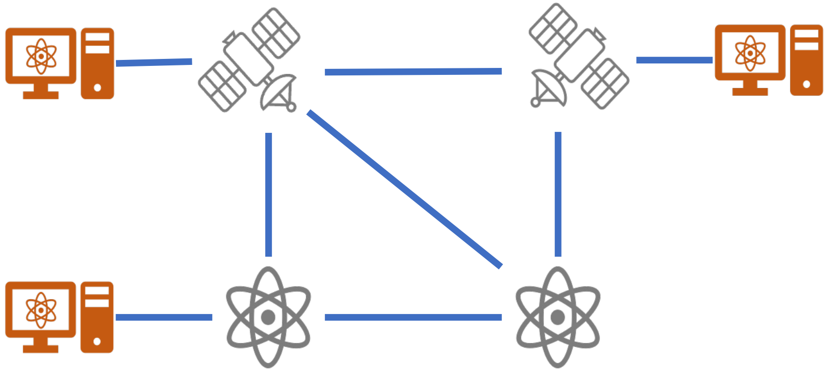
$$\mathcal{E}_e(\rho) = \sum_k \theta_{ek}\sigma_{ek}\rho\sigma_{ek}$$

Problem Definition

Input	Output	Constraint
Network: $G = (V, E)$		
Node partition: $V = V_E \cup V_I$	Estimator $\hat{\theta}_e$ for $e \in E$	Measurements in V_E

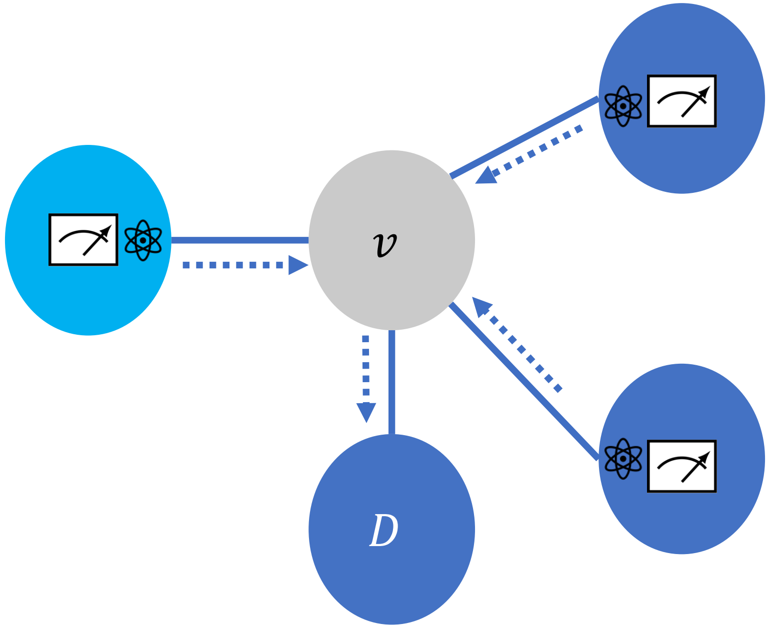


$$\widehat{\mathcal{E}_e}(\rho) = \sum_k \widehat{\theta_{ek}} \sigma_k \rho \sigma_k$$



Quantum Network Tomography as Estimation

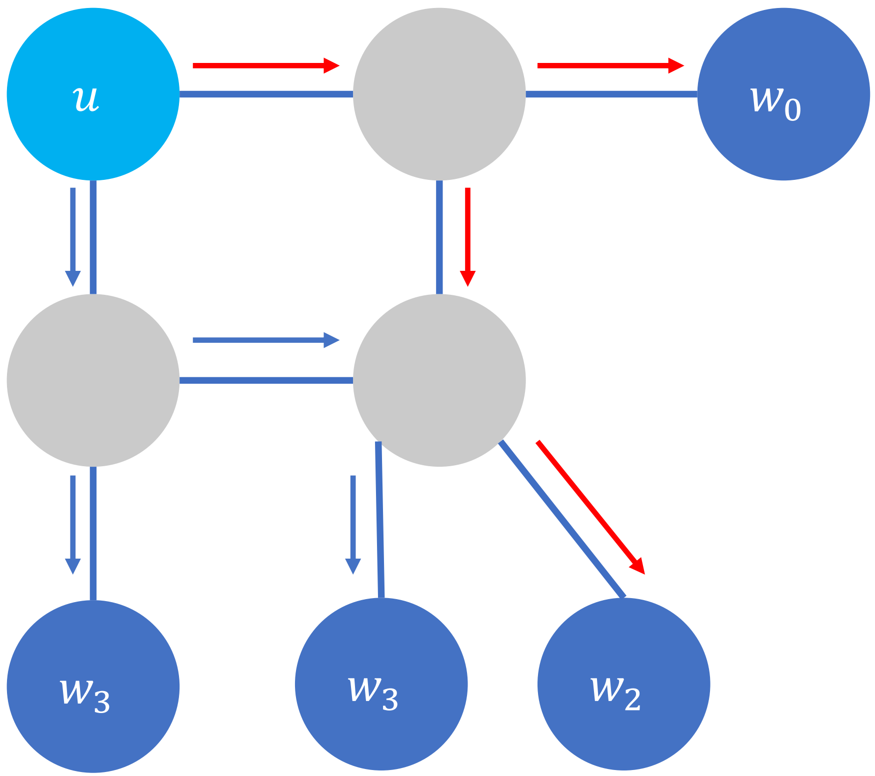
- Parametrization
  - State distributed among end-nodes
  - Mixed state depending on parameters $\rho(\theta)$
- Measurements
  - End-nodes measure each distributed state
  - Outcomes depend on θ
- Parameter estimation
  - Data sent to fusion center
  - Inverse problem yields $\hat{\theta}$



Parametrization as State Distribution

Use network for estimation

- State preparation for rooted trees of G
- Transmission from root to leaves
- Parameter-dependent mixed state
- Characterize links in tree
- Graphs covered by trees



Remarks

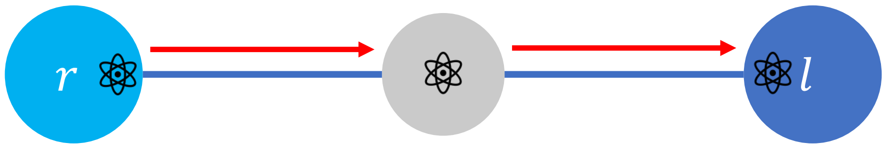
- Trees generalize paths
- Compatible with one-way, two-way architectures

Understanding Parametrization

State distribution is

Preparation of quantum states in end-nodes through network

$$\mathcal{E}_e(\rho) = \theta_e \rho + (1-\theta_e) X \rho X$$



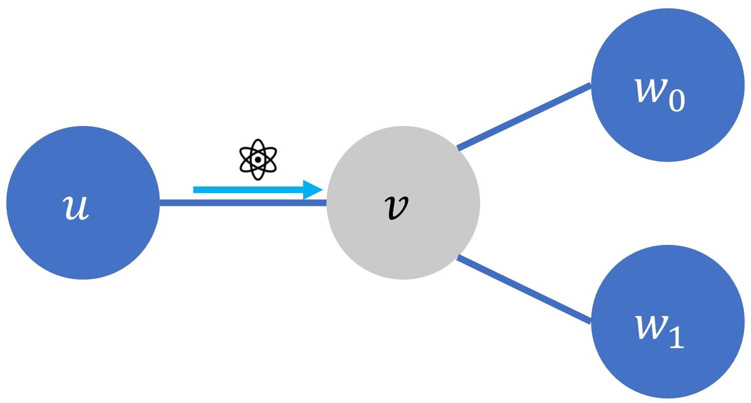
$$\rho_0 = |0\rangle\langle 0|$$

$$\rho_1 = \theta_0 |0\rangle\langle 0| + (1-\theta_e)|1\rangle\langle 1|$$

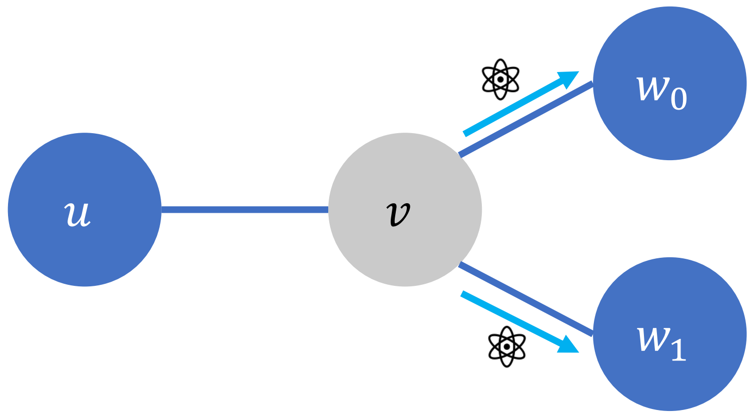
$$\rho_2 = [\theta_0\theta_1 + (1-\theta_0)(1-\theta_1)]|0\rangle\langle 0| + [\theta_0(1-\theta_1) + \theta_1(1-\theta_0)]|1\rangle\langle 1|$$

Node Operations for Distribution

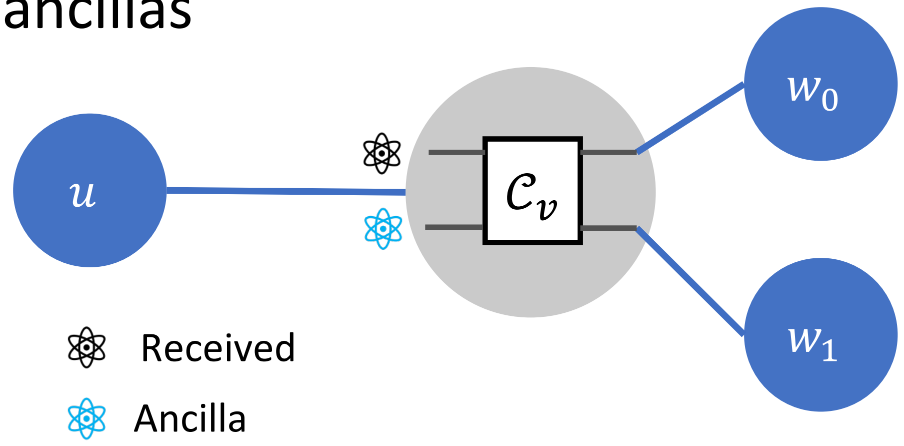
v receives qubit from node u



v sends outputs to neighbors

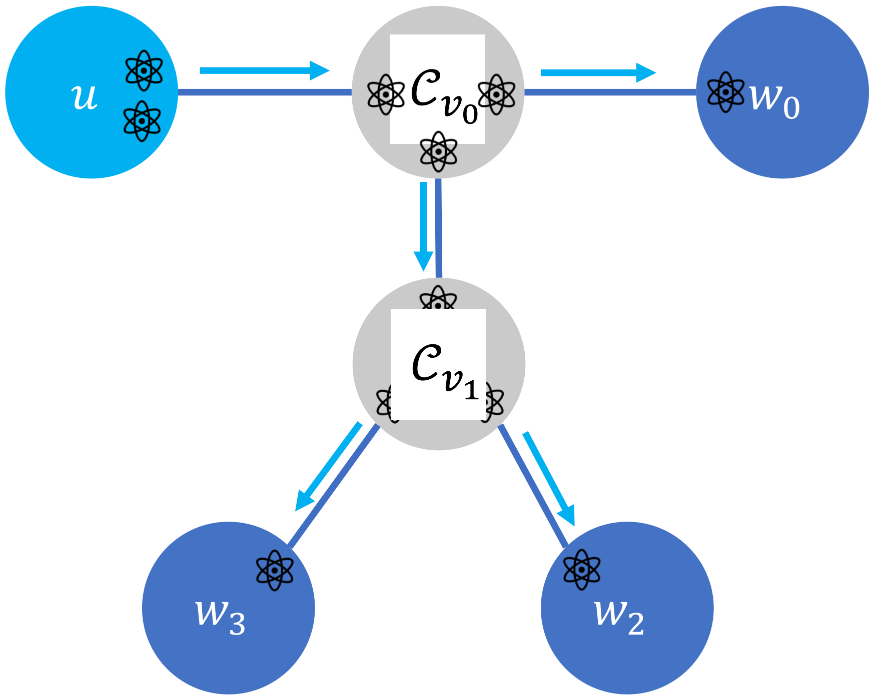


v applies circuit $\mathcal{C}_v$ on received qubit + ancillas



- Generic procedure based on $\mathcal{C}_v$
- Mapping qubits to neighbors is flexible
- Single qubit transmitted for distribution
- No qubits remain in intermediate nodes

Multi-party State Distribution Process

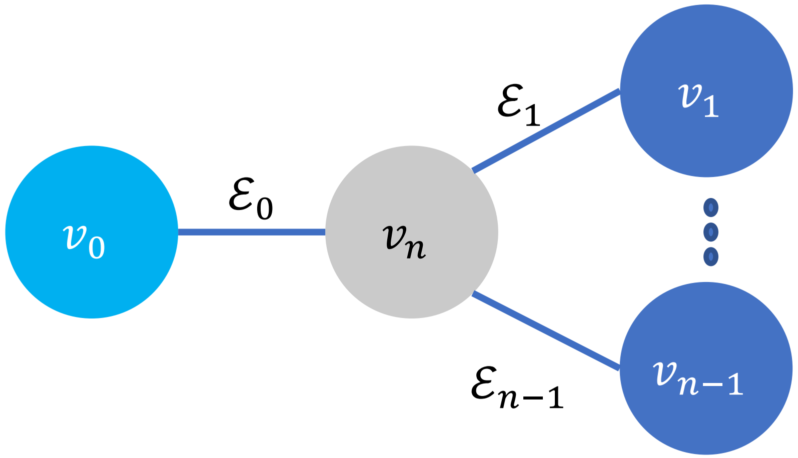


Procedure

1. Prepare qubits at r
2. Transmit qubits to downstream neighbors
3. Apply node operation
4. Repeat 2-3 until there are no more downstream neighbors

Output: Final state $\rho(\theta)$

Quantum Switch Tomography



$\mathcal{H}^2$ qubit Hilbert space, $\rho: \mathcal{H}^2 \to \mathcal{H}^2$

$$\mathcal{E}_e(\rho) = \theta_e \rho + (1 - \theta_e) X \rho X$$

$$|\Phi_s^b\rangle = (|0s\rangle + (-1)^b |1\bar{s}\rangle)/\sqrt{2}$$

$$s = s_1 \dots s_n \in \{0,1\}^{n-1}, b \in \{0,1\}$$

e.g. $|\Phi_1^1\rangle = (|01\rangle - |10\rangle)/\sqrt{2}$

$$\Phi_s^b = |\Phi_s^b\rangle\langle\Phi_s^b|$$

Definitions

- Trees with hop distance 2
- Single-Pauli channels
- Bit-flips for exposition
- 4-node star for exposition

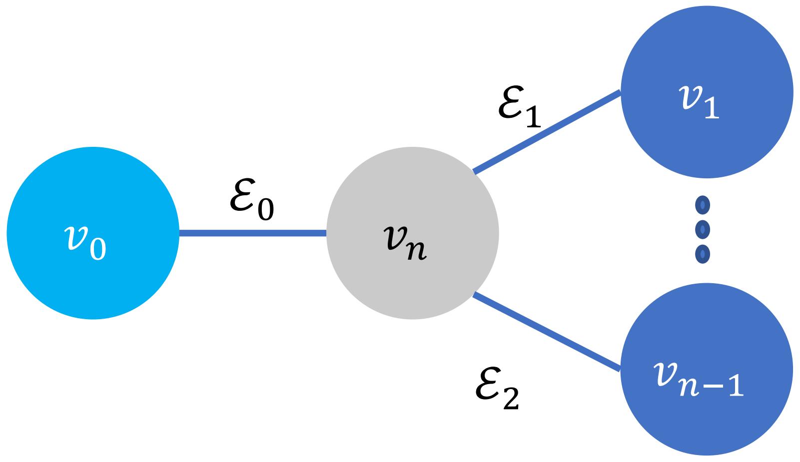
Protocols

- Separable vs entangled state distribution
- Similar distribution algorithms

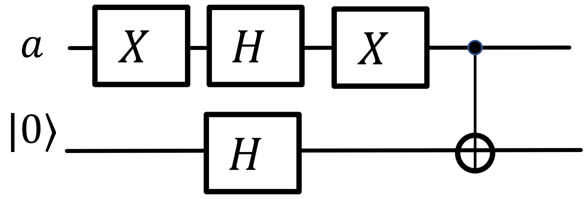
State Distribution and Measurements

Procedure

1. Root prepares state $|\Phi_0^0\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$
2. Root transmits qubit to switch (center)
3. Switch applies tomography circuit
4. Switch sends qubits to leaves
5. Leaves measure in GHZ basis



Tomography circuit



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State Evolution Throughout Distribution

Preparation

$$\rho_0 = \Phi_0^0$$

Transmission through $\mathcal{E}_0$

$$\rho_1 = \theta_0 \Phi_0^0 + (1 - \theta_0)\Phi_1^0$$

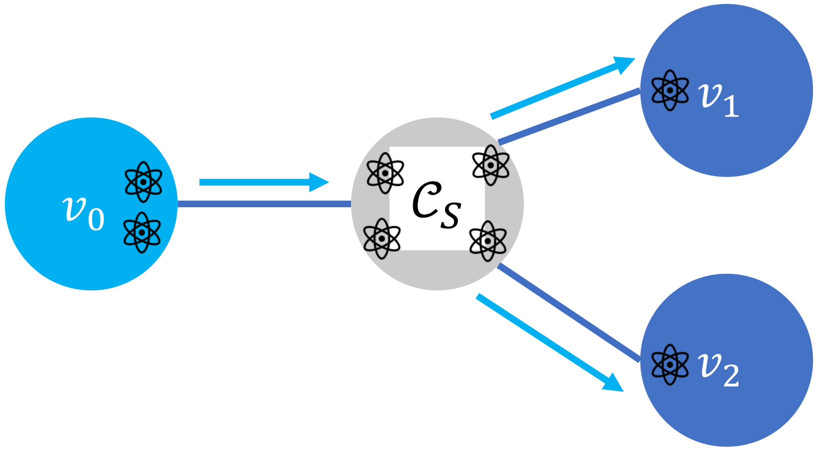
Switch circuit output

$$\rho_2 = \theta_0 \Phi_{00}^0 + (1 - \theta_0)\Phi_{00}^1$$

Transmission to leaves

$$\rho_3 = \sum_{s_k, b \in \{0,1\}} p_E(b, s_1, s_2)\Phi_{s_1 s_2}^b$$

$p_E(b, s_1, s_2)$: GHZ measurement prob.



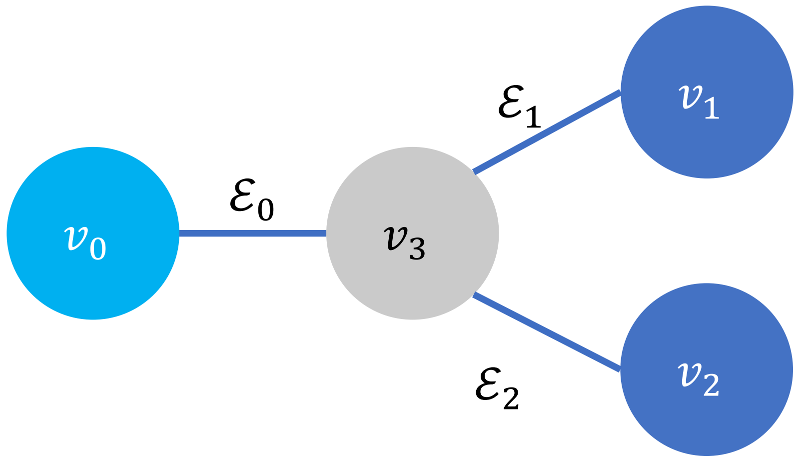
Density Matrix

Diagonal on GHZ basis

$$\rho_3 = \sum_{s_k, b \in \{0,1\}} p_E(b, s_1, s_2) \Phi^b_{s_1 s_2}$$

b	s	state	$p_E(b,s)$
0	00	$\|000\rangle + \|111\rangle$	$\theta_0\theta_1\theta_2$
0	01	$\|001\rangle + \|110\rangle$	$\theta_0\theta_1(1-\theta_2)$
0	10	$\|010\rangle + \|101\rangle$	$\theta_0(1-\theta_1)\theta_2$
0	11	$\|011\rangle + \|100\rangle$	$\theta_0(1-\theta_1)(1-\theta_2)$
1	00	$\|000\rangle - \|111\rangle$	$(1-\theta_0)\theta_1\theta_2$
1	01	$\|001\rangle - \|110\rangle$	$(1-\theta_0)\theta_1(1-\theta_2)$
1	10	$\|010\rangle - \|101\rangle$	$(1-\theta_0)(1-\theta_1)\theta_2$
1	11	$\|011\rangle - \|100\rangle$	$(1-\theta_0)(1-\theta_1)(1-\theta_2)$

Estimators



Entangled state with GHZ measurements

S_j: r.v. for measuring s_j in GHZ, $j > 0$

B: r.v. for measuring b in GHZ

$S_j = F_j \quad \theta_j = p_j \quad B = F_0 \quad \theta_0 = p_0$

Definitions

F_j: r.v. for flip at channel j

$|\Phi_s^b\rangle = (|0s_1s_2\rangle + (-1)^b|1\overline{s_1s_2}\rangle)/\sqrt{2}$

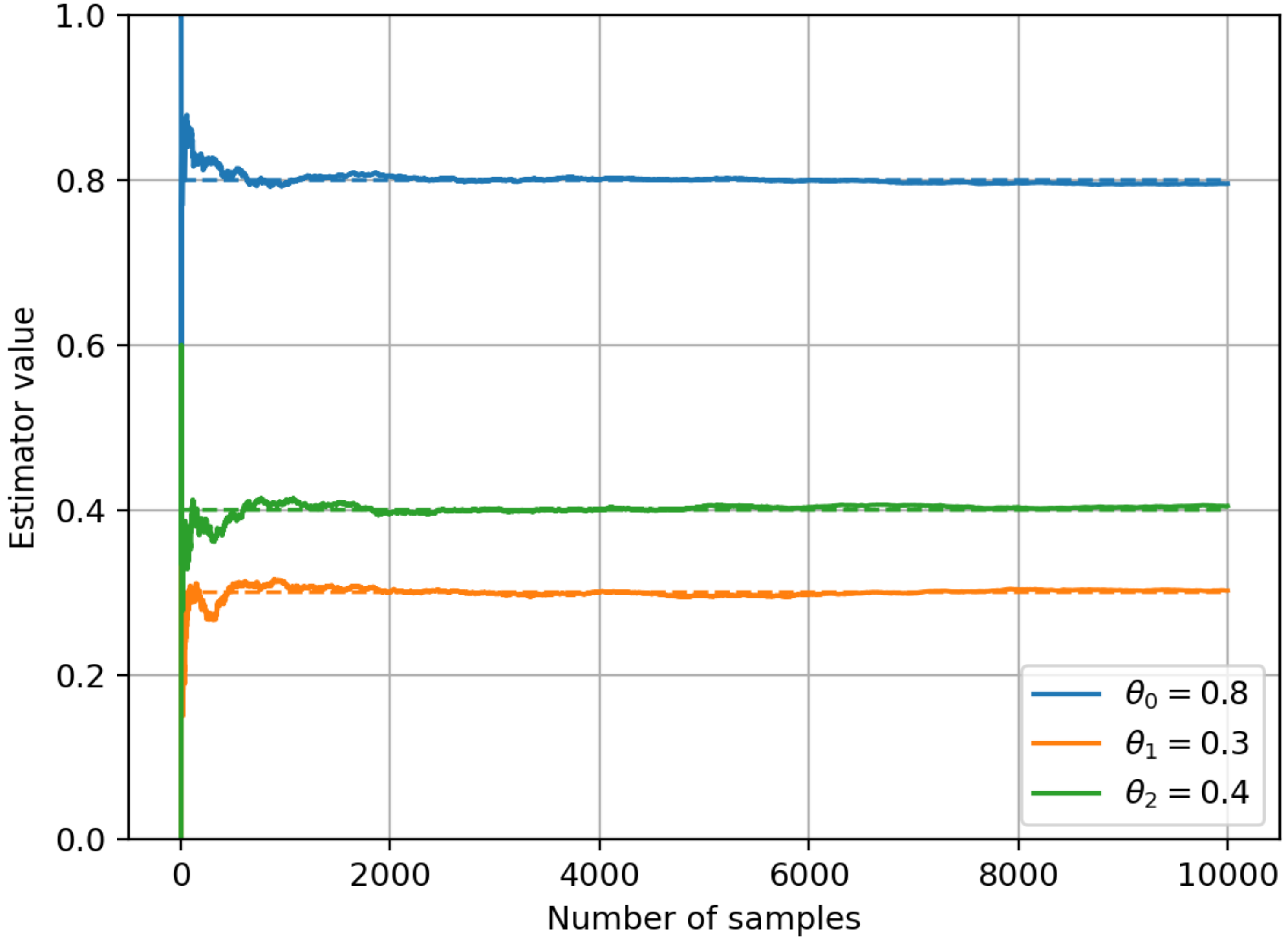
$\mathcal{E}_j(\rho) = \theta_j\rho + (1 - \theta_j)X\rho X$

p_j: prob of outcome 1 in qubit j

Remarks

- Global measurements improve efficiency
- Entanglement not required for ident.
- Twice as many samples needed

Numerical Results



Conclusion

Remarks

- Quantum network tomography
  - Channel parameter estimation in quantum network
  - Captures network characterization from end-to-end perspective
- Estimators for the star can indicate entanglement advantage

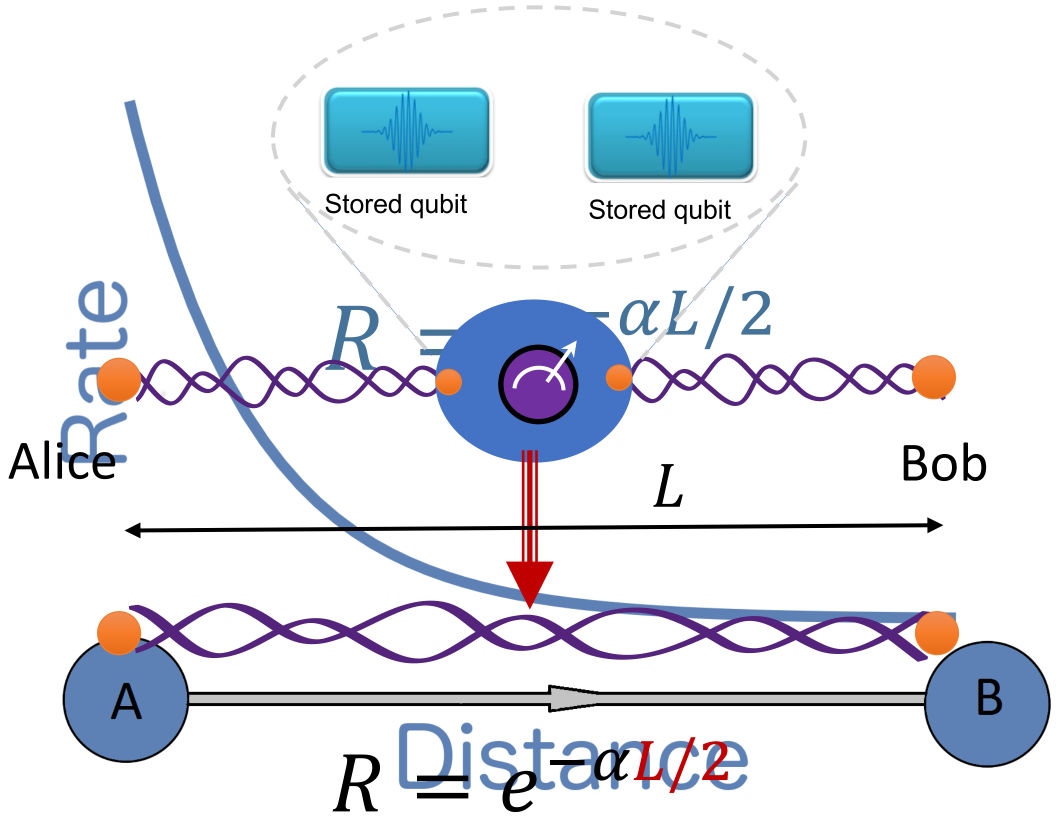
Open problems

- What are the optimal estimation strategies for stars?
- How to generalize estimators for arbitrary trees?
- How to partition network in trees for estimation?
- How do bipartite and multipartite compare?
- Under which conditions entanglement provides advantage?
- Under which conditions are trees identifiable?
- How to generalize efficient estimators for Pauli channels?

Summary and Challenges

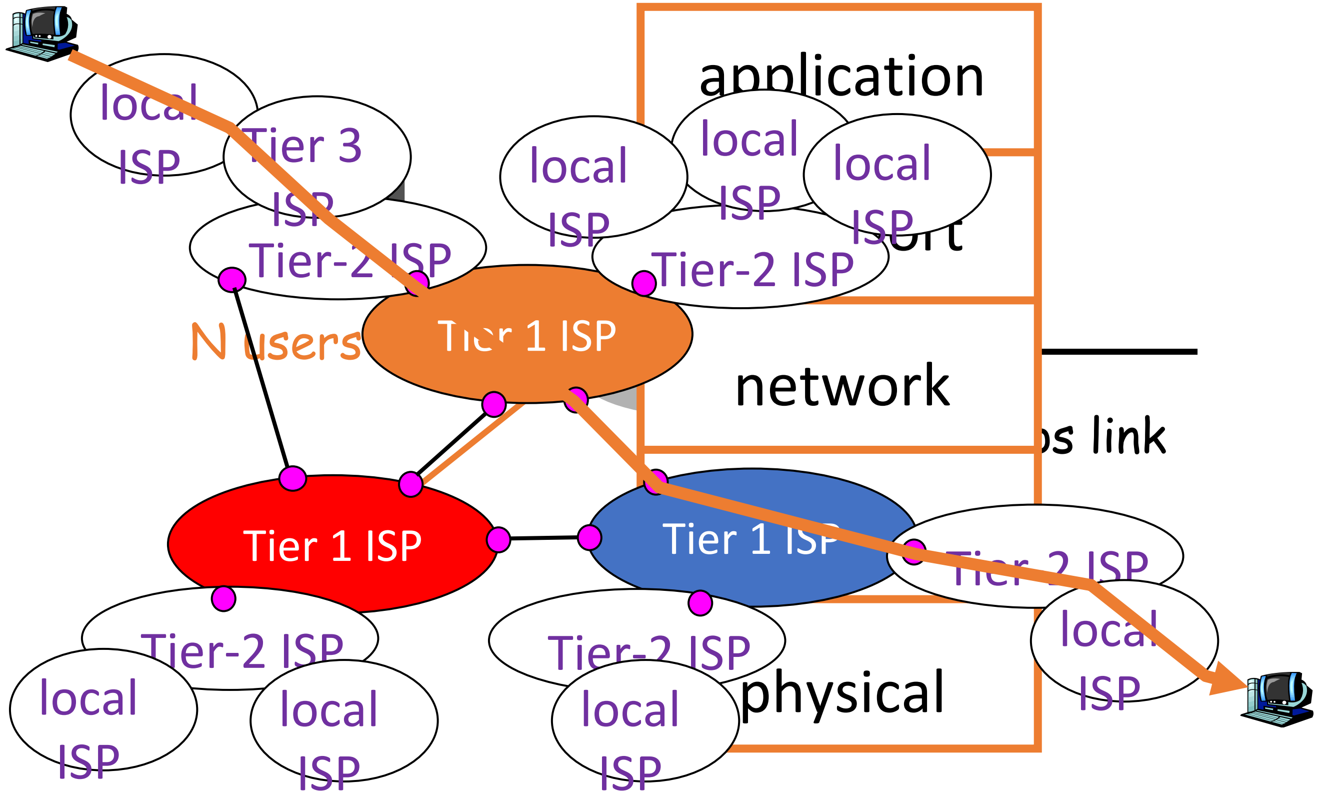
What Quantum Brings to the Table

- Rate decays exponential with distance in fiber
- The non-cloning theorem
- Quantum repeaters
  - Two-way vs one way
- Quantum information is fragile
  - QEC and Distillation
- NISQ era: noisy hardware



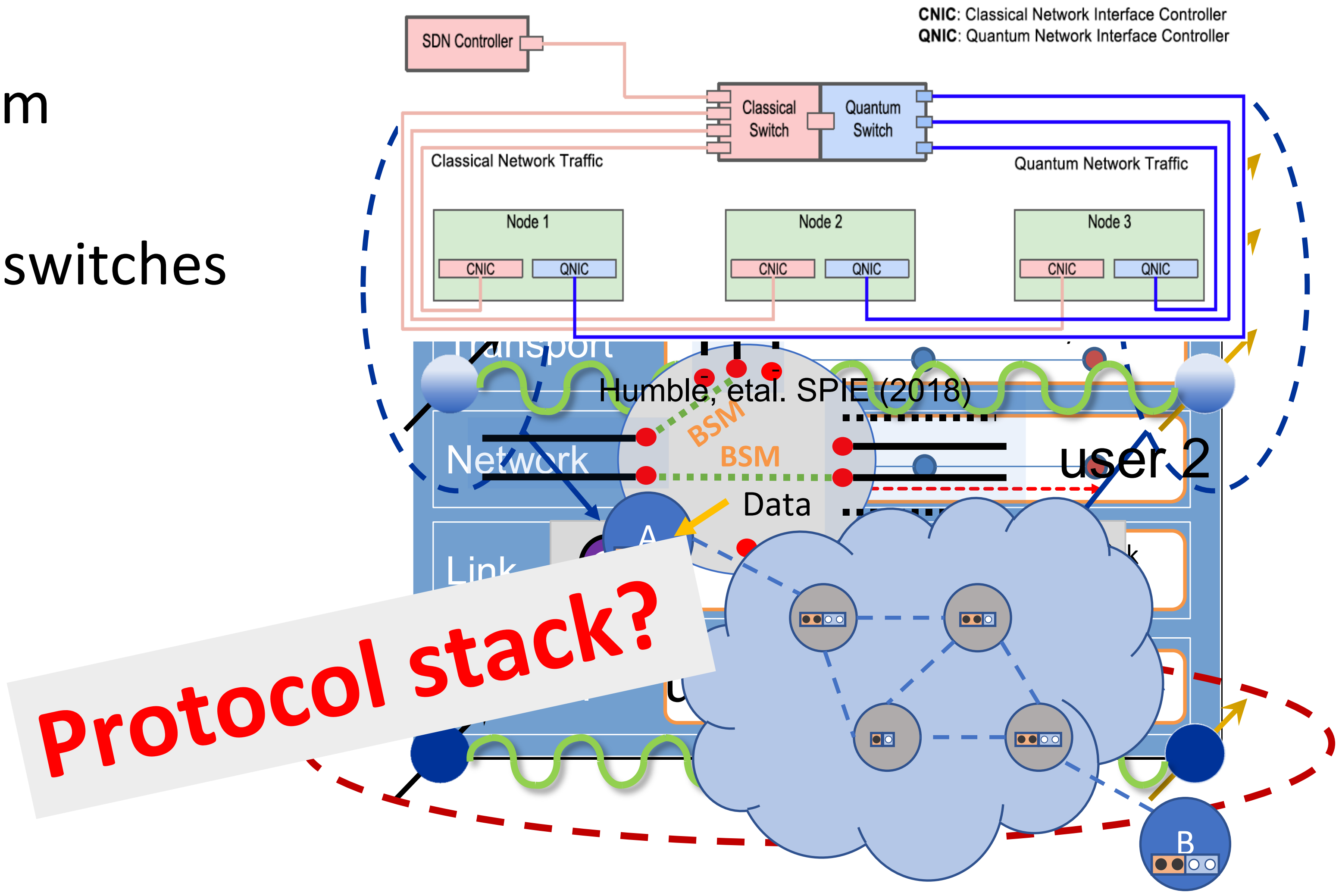
Classical Networks

- Packet vs circuit switching
- Layered design – protocol stack
- Store and forward
- Routing and resource allocation
- Network of networks



Quantum Networks

- One- vs two-way quantum communication
- Quantum repeaters and switches as building blocks
- Mitigating noise
  - In memory
  - In transmission



Guedes de Andrade, etal. *IEEE QCE* (2021)

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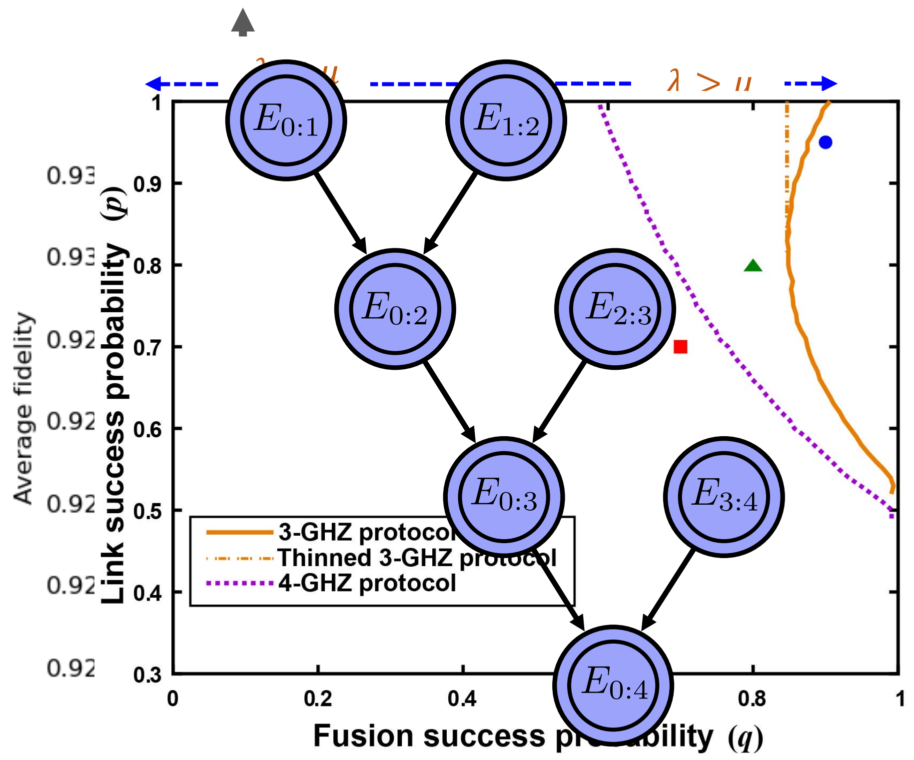
Quantum Networks: Challenges

- Designing efficient, scalable quantum repeaters
- Quantum interconnects
- Layer structure for protocol stack
- Efficient QEC protocols

Capacity and resource allocation

- Network capacity and stability
- Scheduling and noise
- Routing improves rate
  - Distance independent rate with GHZ measurements
- Scheduling improve rates
  - Polynomial decrease with distance for chain topologies
  - Optimization formulation for general topologies



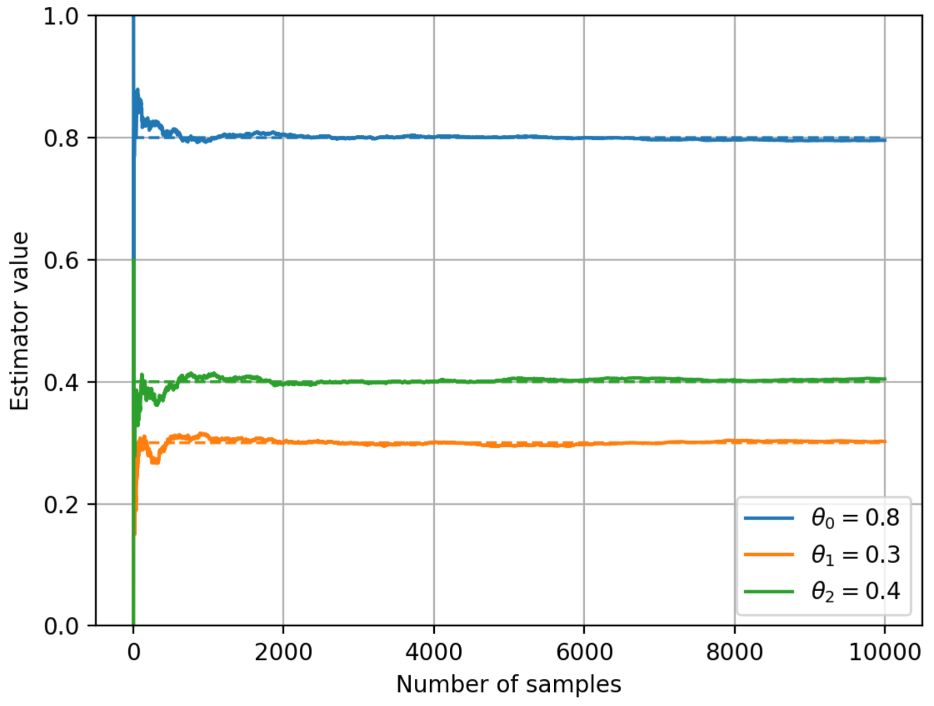
Allocation and Capacity: Challenges

- Adding noise and purification to capacity definition
- Routing in noisy environments
- Scheduling policies for generic topologies and multipartite states
- Optimal purification scheduling
- Optimal buffer management policies for general topologies

Management and Tomography

- Link parameter estimation from end-to-end measurements
- End-nodes communicate through trees
- Identifiability for stars with single Pauli channels
  - Entanglement improves efficiency
  - Not required for identifiability

Entangled state with GHZ measurements



Management and Tomography: Challenges

- Identifying parameters in stars with arbitrary Pauli channels
- Identifiability results for general trees
- Optimal covering of networks with trees
- Loss-resilient tomography protocols

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Thank you!

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Course Evaluation Survey

We value your feedback on all aspects of this short course. Please go to the link provided in the Zoom Chat or in the email you will soon receive to give your opinions of what worked and what could be improved.

CQN Winter School on Quantum Networks

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